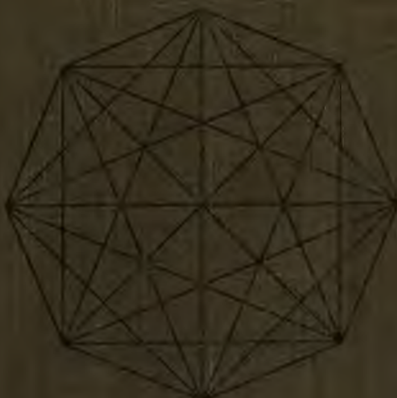
Schoch

Introduction to Geometry



Allyn & Bacon

John A O'Keefe III

1927

INTRODUCTION TO GEOMETRY

A

MANUAL OF EXERCISES FOR BEGINNERS

BY

WILLIAM SCHOCH

CRANE MANUAL TRAINING HIGH SCHOOL, CHICAGO

Boston

ALLYN AND BACON

1904

Norwood Press
J. S. Cushing & Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

PREFACE.

THERE is a well-defined demand for the teaching of the elementary facts and concepts of geometry preceding the formal study of the subject in the high school. But the various attempts to satisfy this demand, as seen in books on grammar school geometry, have not met with success in practice.

The writer believes that the trouble lies not in the subject, but in the method of presentation. If the teaching conformed more to the methods of arithmetic, if the start was made with problems within the experience of the child, and the teachers bent their energies to seeing that the child was forming proper images, then he would be better able to grasp the mathematical relations he is required to learn. The teacher should present to the learner a series of opportunities or problems to enable him to pass from the individual notions of his experience to the clear and definite notions of the science. These problems should be so graded and stated that the pupil's progress will be one of self-development. Every idea, every rule, which he has thus worked out and learned to define for himself, should be constantly recalled and used in the solution of new problems, and so in this way firmly impressed upon his memory.

The purpose of these lessons — which have been tested in the schoolroom — is to suggest such a series of opportunities, aiming to lead the child to a permanent acquisition of the elementary geometric concepts and to enable him to reconstruct them with full consciousness of their

content. For this reason these lessons offer a book of problems rather than a text-book in the ordinary sense of the word. The pupil is to convince himself of geometric truths through measuring and drawing, and thus gradually to come into possession of the material with which formal geometry deals. Lacroix said almost a century ago, "The operation of drawing and of measuring cannot fail to be pleasant, leading the pupils, as by the hand, to the science of reasoning."

Along with this purpose, the practical side of the subject is also taken into account, the child's knowledge being tested, as he goes on, by his power to apply it. All that is taught him is connected with his ordinary everyday experience; and the attempt is made to lead him, in acquiring new knowledge, to turn to use what he has already learned.

The writer acknowledges with thanks his indebtedness for assistance in revising the manuscript and for valuable suggestions on the work to Dr. George A. Bacon, Mr. Charles A. Lowry, Assistant Superintendent of Chicago Public Schools, and Mr. J. A. Foberg of the English High and Manual Training School.

W. S.

MARCH, 1904.

CONTENTS.

INTRODUCTION TO GEOMETRY.

CHAPTER I.

THE CIRCLE.

Exercise 1.

1. A horse is tied to a stake on a grass plot by a rope 36 feet long. What is the shape of the ground that he can feed over? Why?

A figure bounded by a line every point of which is the same distance from a point, the *centre*, is called a *circle;* and the bounding line is called the *circumference.*

2. A dozen children are running around a pole, each keeping the same distance from the pole. What is the shape of the line represented by their tracks?

3. How may a gardener use a stick, or a string, to lay out a circular piece of ground?

4. Make use of these ways:
(i) In drawing circles on the blackboard;
(ii) On paper.

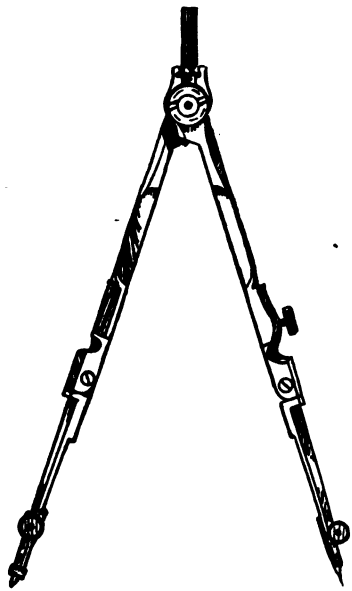


FIG. 1.

5. How can you obtain a perfect circle with the *compasses?* (Fig. 1.) Why?

Exercise 2.

1. Draw a circle and connect a number of points in the circumference with the centre, as shown in Fig. 2. Compare Figs. 2 and 3. In the circle, what do you find corresponds to the rim of the wheel? To the hub? To the spokes?

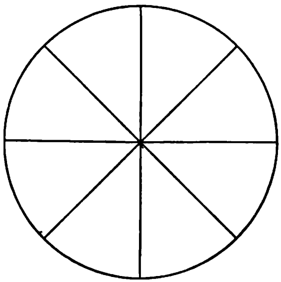


Fig. 2.

Fig. 3.

We call the line which corresponds to a spoke a *radius*, the Latin word for *spoke*.

2. Mention a word that reminds you of *radius*. How are the two related in meaning?

Exercise 3.

1. What line measures the length of the longest straight stick that can lie wholly on a round table?

2. Draw several circles, representing different round tables, and locate the lines which "measure them across."

These lines are called *diameters*.

3. What is meant by a 28-inch wheel? A 6-inch pipe?

Exercise 4.*

1. Draw a picture of a bow with the string fastened to both ends (Fig. 4). What kind of a line is made by the bow? By the string?

It is this resemblance to a stretched string which, presumably, has given the *straight line* its name. In fact, the word *straight* originally meant *stretched.* The ancient Egyptians made a straight line by stretching a string, a method still employed by carpenters.

FIG. 4.

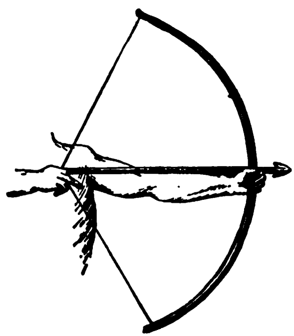


FIG. 5.

2. Notice the picture of the bow (Fig. 5) where the arrow is placed and ready to let go. What change is there in the shape of the bow from the preceding picture?

The bow, or *arc* (from *arcus*, the Latin word for *bow*), when thus bent, is part of a circular line; so the word *arc* has come to stand for the name of such a part. Likewise, since the Latin word for string was *chorda*, the word *chord* has been retained as the name of the straight line joining the ends of an arc.

* TO THE TEACHER. The history of man is full of interesting accounts of his skill in the use of the bow and arrow; and prominent among these we have the story of the Home-coming of Ulysses. Tell it in connection with this exercise.

3. What qualities would you look for in a good bow? In a good string? Why should the arc be longer than its chord? Examine the picture of the bow in Fig. 6. Do you think it is drawn correctly? Why?

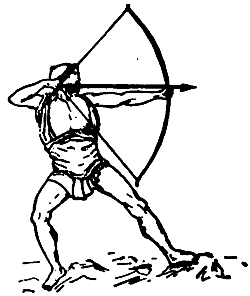


FIG. 6.

Exercise 5.

1. Cut out a circular piece of paper and fold it on one of its diameters. State what you notice concerning the circumference. Will this be true if you fold the circle over any other diameter?

2. Make a statement* about the radii of a circle, and prove it.

3. If you know the radius of a circle, is it necessary to measure the circle to find its width? Why?

4. Make a circle and place points at different distances from the centre less than the radius. What is their position with reference to the circumference? Make a statement concerning the points whose distances from the centre of a circle are less than the radius.

5. Make a statement concerning the points whose distances from the centre of a circle are more than the radius, and explain it by a drawing.

6. How many points can you locate 3 inches from a fixed point? What is the quickest way of locating all of them? Why?

7. Two points, like A and B, are 3 inches apart. A third point, X, is so situated that it is

A · · · · · · · · B

* TO THE TEACHER. Insist on full and clear statements. The best test of knowledge is ability to express it.

$2\frac{1}{2}$ inches from A and 2 inches from B. Make a drawing to locate X.

(*a*) How many positions for X are possible?

(*b*) What must be the distance between A and B, so there can be but one position for X? (Two answers.)

Points are named by using capital letters.

8. Select any point on your paper and name it P.

(*a*) How many circles can you draw with P as a centre and with a radius equal to 1 inch?

(*b*) How many circles with a radius of 1 inch can you draw whose circumferences pass through P? Draw one. Draw another. How are the centres of these circles located with respect to P? Prove your statement.

Exercise 6.

1. What is there about the circle that makes it a suitable symbol for the seasons? The hour? The year?

Note the expressions, "the round of the year," "the circle of events."

2. The use of the circle to symbolize time is very old. We find it in the Zodiac (Fig. 7), a picture of the year as conceived by the ancient Chaldeans, showing the apparent path of the sun about the earth in 360 steps, the number of days they counted to their year. Historians * mention this fact to account for the apparently awkward division of the circumference of the circle into 360 parts, or *degrees*, a word originally meaning steps.

What, then, is a degree?

The degree is subdivided as follows:

1 degree = 60 minutes;
1 minute = 60 seconds.

* Madame Ragozin, *The Story of Chaldea;* M. Cantor, Vol. I, p. 639; Scaife, *Smithsonian Report for 1889*, p. 753.

The sign for degree is (°); and (′) and (″) stand for minutes and seconds, as first and second subdivisions of a degree.

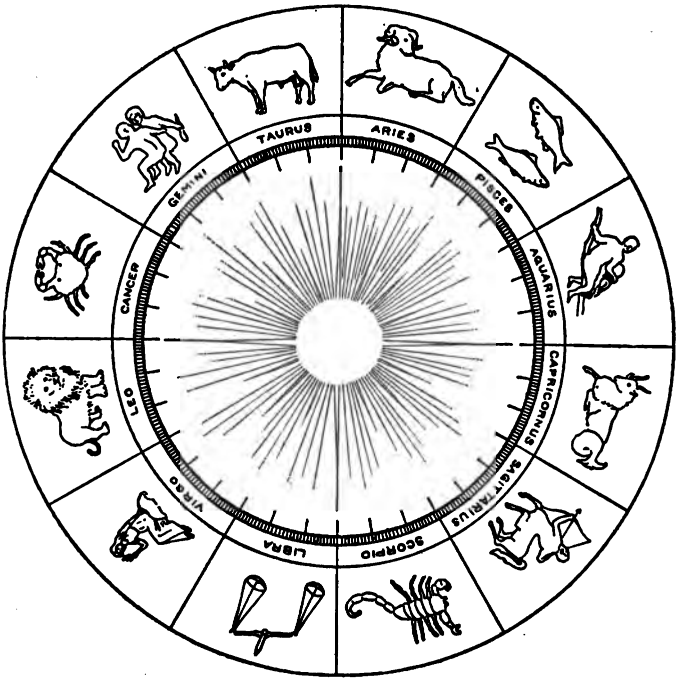


FIG. 7.

3. What division of the circumference of a circle and what subdivisions would be more in harmony with our number system?

Exercise 7.

1. According to the Chaldean view, how many degrees will the sun be from its annual starting point at the end of 45 days? 90 days? 180 days? 270 days? 360 days?

The course is opposite to the direction in which the hands of a clock move, beginning with the space Aquarius, which corresponds to our month of January, through Pisces, our February, and on through Aries, Taurus, etc.

2. Cut a circular piece of paper having a radius of 3 inches, and fold it into 8 equal parts. Crease well and open. Then draw on this paper — using the same centre — two other circles, one with a radius of 1 inch, and the other with a radius of 2 inches. Compare the three arcs in any one of the eight sections of the circle. How are they related in length? How are their halves related in length? Their fifths? Their ninths? The spaces called degrees? What statement can you make concerning equality of degrees in different circles? In the same circle?

3. A and B ran a race on a circular track. They started from the same point and went in opposite directions. When they met, A had covered $\frac{7}{15}$ of the length of the track. Find the number of degrees each one passed over.

4. Find the length of an arc of 30° in a circle whose circumference is 60 inches.

HINT. What part of the whole circumference is this arc?

5. Considering the circumference of the earth at the equator to be 25,000 miles, what is the distance between two places on the equator which are 50° apart? 90° apart? 135° apart?

6. If a ship sails 1500 miles on the equator, through how many degrees will it pass?

Exercise 8.

1. Through how many degrees does a point on the equator revolve in 1 hour? Through what part of a degree in 1 minute of time? How far in 1 second of time?

2. A lives 90° east of B. How much sooner will A see the sun rise than B?

3. If two men live $7\frac{1}{2}°$ apart, east and west, what is the difference in their time?

4. A difference of 1° in location east and west makes what difference in time? Of 1′? Of 1″?

Difference in location east and west is called *difference in longitude*. This term originated with the early geographers. They only knew a part of the world, and that was longer from east to west than from north to south. So naturally they considered distance to the east and west as *length*, or *longitude*, from the Latin word for *length*, and distance to the north and south as *breadth*, or *latitude*, from the Latin word for breadth.

5. What is the difference in time, when the difference in longitude is 65° 17′ 45″?

SOLUTION.

65° correspond to $65 \times \frac{1}{15}$ hr.	= 4 hr. 20 min.		
17′ correspond to $17 \times \frac{1}{15}$ min. =		1 min.	8 sec.
45″ correspond to $45 \times \frac{1}{15}$ sec. =			3 sec.
Difference in time =	4 hr.	21 min.	11 sec.

6. What is the difference in time when the difference in longitude is:

(*a*) 22° 15′?
(*b*) 48° 30′ 15″?

7. What is the difference in longitude, when the difference in time is 2 hr. 19 min. 10 sec.?

SOLUTION.

2 hr. of time correspond to $2 \times 15°$ =	30°		
19 min. of time correspond to $19 \times 15'$ (or $\frac{1}{4}°$) =		4° 45′	
10 sec. of time correspond to $10 \times 15''$ (or $\frac{1}{4}'$) =			2′ 30″
Difference in longitude =		34° 47′ 30″	

8. What is the difference in longitude, when the difference in time is:

(*a*) 4 hr. 10 min. 30 sec.?

(*b*) 54 min. 30 sec.?

9. What is the difference between the longitudes and between the times of:

(*a*) Ann Arbor, Mich. (83° 43′ 48″ W), and Cambridge, Mass. (71° 7′ 45″ W)?

(*b*) New York (73° 58′ 25″ W) and Chicago (87° 36′ 42″ W)?

(*c*) New York and San Francisco (122° 25′ 41″ W)?

(*d*) New York and Paris, France (2° 20′ 15″ E)?

With the development of railroads, the inconvenience arising from differences of time between distant places made necessary some uniform system, and in 1883 what is known as "Standard Time" was adopted. It has now come into general use, furnishing a satisfactory solution of the difficulty.

This system divides the continent into four sections, each extending over fifteen degrees of longitude. The time of each section is that of its central meridian, which is always a multiple of 15° from Greenwich. "Eastern Time" is that of the 75th meridian west from Greenwich, "Central Time" that of the 90th, "Mountain Time" that of the 105th, and "Pacific Time" that of the 120th.

10. Find the differences in standard time for the places mentioned in (*a*), (*b*), (*c*), and (*d*) of problem 9.

11. Is your school time local or standard? What kind of time is maintained by the Western Union clocks? How is it regulated?

12. What is the greatest difference possible between standard and local time?

CHAPTER II.

THE ANGLE.

Exercise 9.

1. Make a picture of:

    (*a*) A ceiling;
    (*b*) A wall;
    (*c*) The cover of a book.

How are the corners represented?

2. Name the different kinds of corners. What kind of a corner is the most common? Draw a picture of a sharp corner. A blunt corner. A square corner.

These pictures of corners are called *angles*, from the Latin word *angulus*, meaning *corner*. We speak of the square corners as *right* and of the others as *not right*, or *oblique*. Accordingly, we have *right angles* and *oblique angles*, the latter being of two kinds: *acute* and *obtuse*. *Acute* and *obtuse* are derived from Latin words meaning *sharp* and *blunt*, respectively.

CORNERS.		ANGLES.	
(1) *Square.*		(1) *Right.*	
(2) *Not square*	Sharp.	(2) *Oblique*	Acute.
	Blunt.		Obtuse.

Exercise 10.

1. How can we distinguish between different angles in regard to size? Are the angles in Chart I all equal? Which is the largest? The smallest? How can you tell?

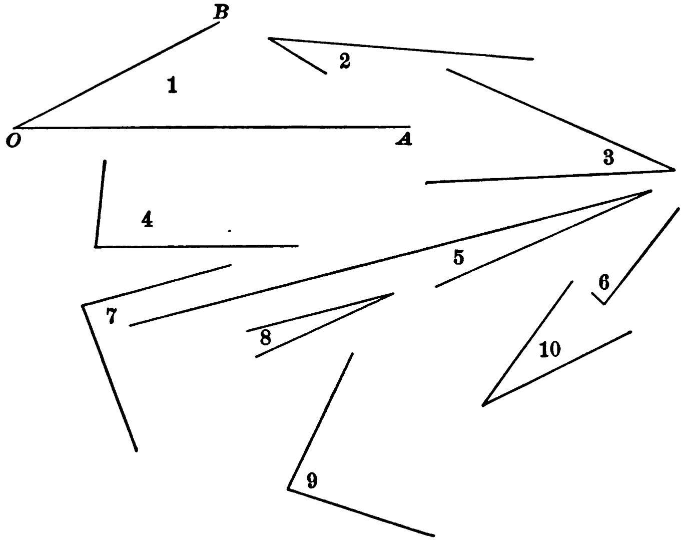


CHART I.

2. Does the size of an angle depend on the length of its sides?

3. Procure a circular card (a clock-dial) with two movable hands. Place the hands so they form an acute angle. What will you have to do to make an angle that is larger? One that is smaller? How can you measure this turn?

We measure an angle by ascertaining through how many degrees one of its arms will have to be turned to fall on the other. The point around which the arm is turned is called *vertex*, the Latin for *turning-point*. To measure the amount of turning, we use an instrument, called the *Protractor*, which is really only a half-circle with its 180 degrees marked on it (Fig. 8).

4. With the aid of the protractor:

(*a*) Measure the angles in Chart I.

(*b*) Draw angles of 90°; 30°; 50°; 110°; 150°.

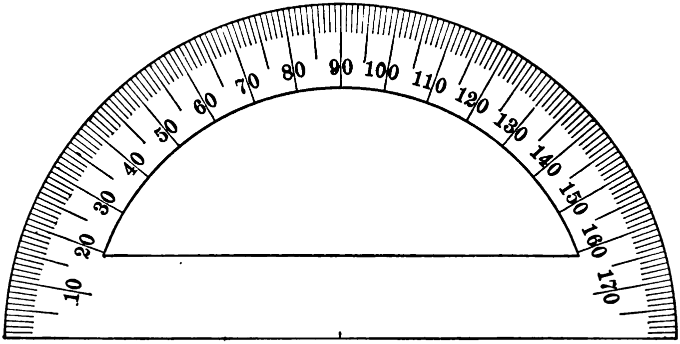


FIG. 8.

Exercise 11.

Angles are named:

(*a*) By placing figures or small letters inside the angles.

(*b*) By lettering the sides forming the angles. Angle *AOB* (see 1 in Chart I) is the angle formed by the lines *AO* and *OB*. Note that in writing the name of an angle the letter at the vertex is placed in the middle.

(*c*) By a single capital at the vertex.

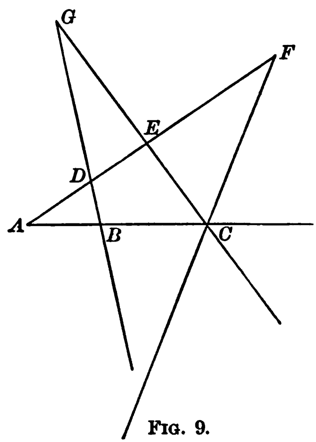


FIG. 9.

1. Name the angles in Fig. 9 in as many ways as you can.

2. Estimate the size of these angles and test with the protractor.

3. Make a diagram by drawing at random a number of straight lines to cross each other as in Fig. 9, and letter all the points of intersection.

Then with the aid of a protractor copy all the angles, placing each one by itself and lettering it the same way as in your diagram.

4. A convenient way of making an angle equal to a given angle, XYZ, is to take a piece of paper with a straight edge, place it on the given angle, the straight edge against YX, and then mark on it both Y, the vertex of the angle, and P, the point where YZ issues from under it (Fig. 10). The angle is thus picked up on the paper and may be reproduced from it in any position. When does this prove a more accurate method of copying an angle than the one with the protractor?

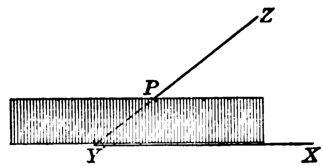


FIG. 10.

Exercise 12.

1. Draw any acute angle AOB. Then double it. When will the result be another acute angle? A right angle? An obtuse angle? Test the angles with a protractor.

2. What will be the result, if an obtuse angle be doubled? Draw a diagram with an angle of 120°. Test with your protractor. Why should you be careful to draw the lines forming the angles very fine?

A *geometric line* has neither breadth nor thickness.

3. With the aid of the protractor, make an angle of 150° and draw a line which will divide it into two equal parts.

This is cutting the angle in two, or *bisecting* it, an expression derived from a Latin word meaning *to cut in two.* The cutting line is called the *bisector.*

Exercise 13.

1. How many degrees are contained in the angle made by the hour and the minute hands of a watch at 1 o'clock? How many degrees in the angle made by the hands of a church-tower clock at the same time?

The angles here are formed by lines presumably passing lengthwise through the middle portions of the hands.

2. How many degrees are passed over by the minute hand in a quarter of an hour? How many are passed over by the hour hand in an hour? In half an hour? In 15 minutes?

3. How large an angle is contained between the hands of a clock at

(*a*) 3?	(*c*) 9?	(*e*) 8?	(*g*) 7?	(*i*) 5?
(*b*) 4?	(*d*) 2?	(*f*) 12?	(*h*) 11?	(*j*) 6?

4. Over how many degrees does the large hand of a clock pass:

(*a*) In 60 min.?	(*c*) In 12 min.?	(*e*) In 6 min.?
(*b*) In 30 min.?	(*d*) In 1 min.?	(*f*) In 2 hr.?

5. What angle is made by the hands of a clock at 11.33?

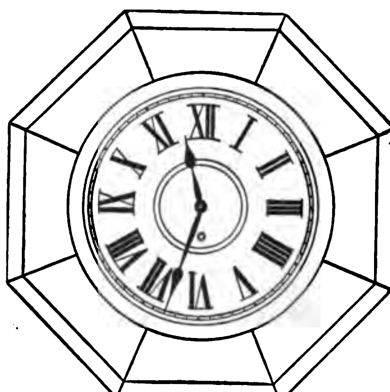


FIG. 11.

SOLUTION. The large hand is on the 33-minute mark and the small one is somewhere between XI and XII (see Fig. 11). Both hands have gone over equal parts of their respective courses since 11 o'clock. (What are their respective courses between 11 and 12? What are the parts gone over?) A course of

33 minute-spaces, or $\frac{11}{20}$ of one revolution of the large hand, corresponds to $\frac{11}{20}$ of the space between the XI and the XII, or a course of $2\frac{3}{4}$ minute-spaces of the small hand. The small hand is therefore on $57\frac{3}{4}$, and the hands are $24\frac{3}{4}$ minute-spaces apart.

An angle between hands 1 minute-space apart $= 6°$.

An angle between hands $24\frac{3}{4}$ minute-spaces apart $= 24\frac{3}{4} \times 6°$

$= 148° \, 30'$.

Ans.

6. What angle is made by the hands of a clock at

(*a*) 12.12?	(*e*) 7.48?	(*i*) 2.06?	(*m*) 4.15?
(*b*) 12.30?	(*f*) 2.24?	(*j*) 3.30?	(*n*) 1.10?
(*c*) 12.48?	(*g*) 7.42?	(*k*) 9.18?	(*o*) 11.38?
(*d*) 6.30?	(*h*) 11.42?	(*l*) 8.20?	

Exercise 14.

1. What angle is formed by the direction-lines of the compass (Fig. 12) through the points:

(*a*) N and E?
(*b*) N and S?
(*c*) E and W?
(*d*) N and NE?
(*e*) S and SW?
(*f*) E and SW?
(*g*) SE and SW?
(*h*) W and ENE?
(*i*) N and ESE?
(*j*) SSW and N?
(*k*) ESE and NW?
(*l*) SSW and ENE?
(*m*) N and NE_bN?
(*n*) SW_bS and N?
(*o*) SW_bS and SW_bW?
(*p*) NE_bE and W_bN?

FIG. 12.

2. Toward what point of the compass is a ship heading, if the angle between the meridian-line and the course is

(*a*) 90° E?	(*e*) 45° E?	(*i*) $157\frac{1}{2}$° W?	(*m*) $56\frac{1}{4}$° W?
(*b*) 90° W?	(*f*) $67\frac{1}{2}$° W?	(*j*) $67\frac{1}{2}$° E?	(*n*) $11\frac{1}{4}$° E?
(*c*) 45° W?	(*g*) $112\frac{1}{2}$° E?	(*k*) 180°?	(*o*) 135° W?
(*d*) 135° E?	(*h*) $22\frac{1}{2}$° E?	(*l*) $33\frac{3}{4}$° E?	(*p*) $157\frac{1}{2}$° E?

The meridian-line is a line from the ship toward the north pole.

Exercise 15.

Take your circular card and place one of the hands at 0° and the other at 90°. Move the hand at 90° gradually to 180°. Observe the corner at the vertex. From being *blunt* it grows into being no corner at all. It becomes *straightened* out. So, from the appearance of its sides as corner-makers, or rather its resemblance to a straight line, the angle of 180° has been named a *straight angle.*

Exercise 16.

1. In putting up a house, how does a carpenter satisfy himself that the floor he is laying is horizontal?

By the use of a spirit-level.

2. What aid has he in building the walls vertical?

A plumb-line.

3. If you rest a spirit-level on a horizontal floor and suspend a plumb-line directly over it, what sort of an angle will be formed? (Fig. 13.)

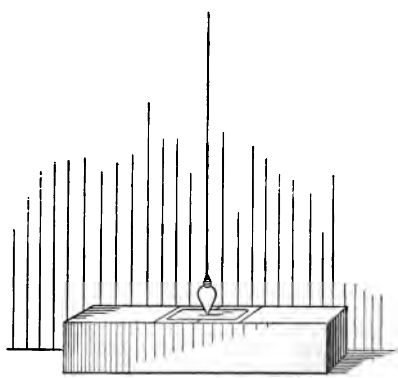


FIG. 13.

This picture was a common sight with our forefathers, when pendulum-clocks were in vogue, so much so that they spoke of a line which, like the plumb-line, forms a right angle with a base as being *perpendicular*, *i.e.* like a perpendicle, another word for pendulum.

4. If one of the sides of a right angle is vertical, what will be the position of the other?

5. If one of the sides of a right angle is horizontal, is the other necessarily vertical? Examine the edges of objects in the room, the outlines of buildings, a brick, and the square wooden block shown in Fig. 14, commonly called a *cube*.

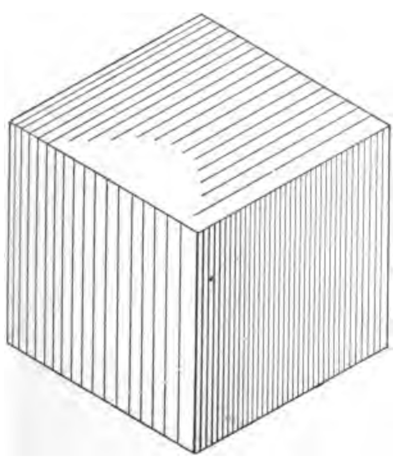


FIG. 14.

In describing the sides of a right angle, meaning *any*, or *every*, right angle, it may, or may not, be correct to say that one side is horizontal and the other vertical. This description would fit some right angles, but not all. So we use the word *perpendicular* to describe the relative position of the sides of a right angle to each other.

6. Draw right angles in at least six different positions. How can you make sure that they are right angles? *

* TO THE TEACHER. Let the pupil feel the need for an instrument, or aid of some kind, to secure correct drawing. Suggest the use of the T-square as an aid in mechanical drawing.

7. Why are the instruments shown in Figs. 15 and 16 also called *Try-squares?*

8. Explain the expression, "just right to a T."

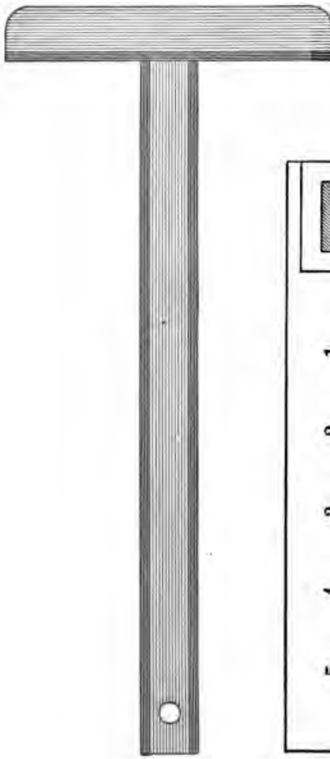


Fig. 15. — T-square.

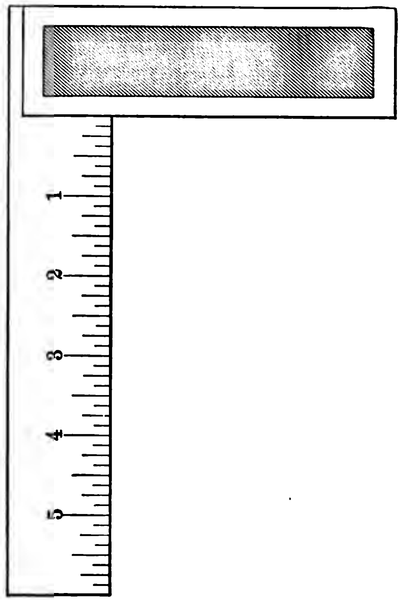


Fig. 16. — Carpenter's Square.

Exercise 17.

1. Take a paper circle and fold it in halves, breaking the lower part over on the upper.

(*a*) What kind of a line in the circle does the crease represent?

(*b*) What kind of an angle?

(*c*) Fold again, from right to left, showing the circle in quarters (Fig. 17). Apply this folded piece of paper to the angles of the cube. What do you find?

(*d*) Can you use this as a square corner, or pattern for making right angles, the same as you do the T-square? Why?

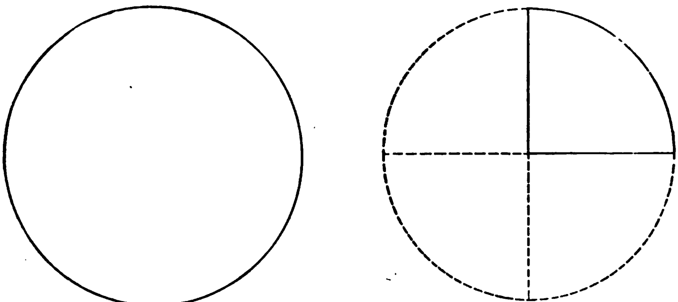


FIG. 17.

2. Open the paper to show the creases, and name the points.

(*a*) How many right angles do you see? Are they equal? Why?

(*b*) Are all right angles equal? Why?

(*c*) How many straight angles do you see?

(*d*) Are all straight angles equal? Why?

3. What is the relation of a straight angle to a right angle?

4. What is the relation of a right angle to an acute angle? To an obtuse angle?

5. Why are not all acute angles equal?

Exercise 18.

1. Make a right-angle pattern, using paper of any shape not circular.

2. With the aid of such a pattern:

(*a*) Erect a perpendicular to a given line at a given point.

(*b*) Drop a perpendicular from a given point to a given line.

3. Fold a piece of paper to make an angle of 45°.

4. What other angles can you make the same way? Give a list of them.

5. Make an angle of $67\frac{1}{2}°$ without using the protractor.

HINT. Combine angles which you can obtain by folding. Note that $67\frac{1}{2} = 45 + 22\frac{1}{2}$.

6. Form likewise angles of

(*a*) 135°; (*b*) $112\frac{1}{2}°$; (*c*) $56\frac{1}{4}°$.

CHAPTER III.

CONSTRUCTION OF PLANE FIGURES.

A. Two Straight Lines: their Relative Positions.

Exercise 19.

1. It takes at least two lines to represent an angle. Can two lines be drawn to represent more than one angle?

2. Draw two lines making only two angles, one of which measures

(*a*) 60°; (*b*) 117°; (*c*) 90°; (*d*) $67\frac{1}{2}$°.

How many degrees does the other angle contain?

3. Is there a way of obtaining these results without actual measurement? Why? Find the values of two such angles, if one is nine times as large as the other.

4. Draw two straight lines to make as many angles as possible. (Lines that "cut each other" in this way are said to be *intersecting*.) If one of the angles be (*a*) 100°, (*b*) 38°, (*c*) 135°, (*d*) 60°, what will each of the other angles be?

When two lines intersect, they form four angles. Taken in pairs, these angles are of two kinds:

(*a*) *Adjacent angles*, angles having the vertex and one side in common.

(*b*) *Vertical angles*, angles having only the vertex in common.

5. Make a statement about a pair of adjacent angles formed by two straight lines, and prove it.

6. Make a statement about a pair of vertical angles.

Exercise 20.

1. Draw angle $AOB = 72°$, and continue line AO to C. Bisect angles AOB and BOC and call their bisectors OD and OE, respectively. What is the size of the different angles formed? (Fig. 18.)

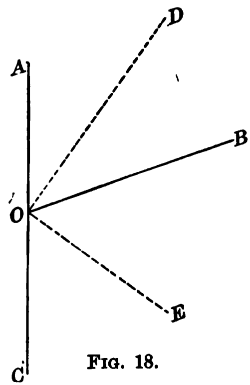


FIG. 18.

2. Repeat this construction with angle AOB equal to

(*a*) 116°; (*b*) 90°; (*c*) 45°; (*d*) 60°.

What is the sum of the angles about the point of intersection? What position relative to each other do the angle-bisectors OD and OE occupy in each case? Is there any reason why that should be so?

3. Draw two intersecting lines, BOY and ROD, O being the point of intersection. Bisect angles ROY and BOD, and call the bisectors OC and OM, respectively. What position relative to each other do these angle-bisectors occupy? Prove your answer, supposing angle ROY to be

(*a*) 30°; (*b*) 80°; (*c*) 45°; (*d*) 90°.

4. There are three angles completely surrounding a point as a common vertex, the second 10° greater than the first, and the third 10° greater than the second. How large is the first? Draw the angles.

5. Around a point as common vertex there are eight angles, each angle 6° greater than the one preceding, counting from the smallest. Draw the angles.

6. A line BR is drawn, passing through the vertex of an angle MAN (Fig. 19), but not through the angle itself.

(*a*) If $\angle MAN$ is equal to 60°, what is the sum of the angles thus formed, $\angle MAB + \angle RAN$?

$\angle$MAN is read "angle MAN."

What is the sum, if $\angle MAN$ is

(*b*) 90°? (*d*) $112\frac{1}{2}$°?
(*c*) 45°? (*e*) n°?

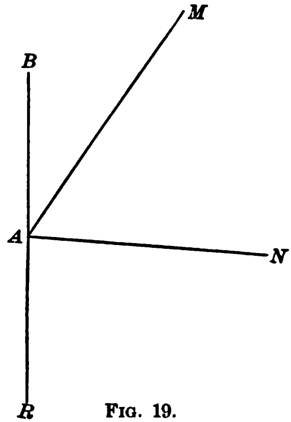


Fig. 19.

Exercise 21.

1. Can you conceive of two lines which will not form an angle, no matter how long they are made or how far they are extended? Examine a railroad track or a street-car track. What must be true of the two lines representing the track?

They must keep an even distance from each other, or run *parallel*, which means "one beside the other," never meeting.

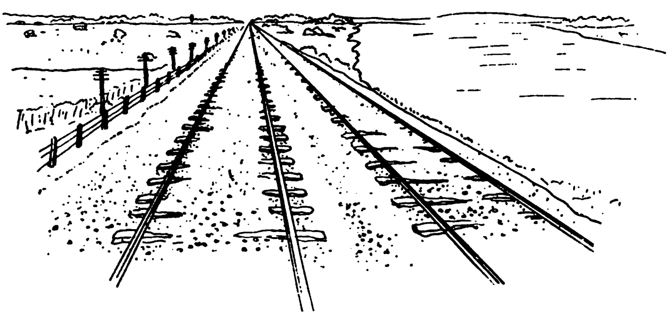


Fig. 20.

2. Why is it that in pictures of railway tracks (Fig. 20) the lines of the rails seem to meet in the distance?

3. When men build a railroad, how do they make sure that the rails run parallel?

4. What is the *gauge?* Distinguish between *narrow gauge* and *standard gauge.*

5. What angle does the line of the gauge form with the parallel lines of the rails?

6. What angle is formed by the lines of the track?

7. Construct two parallel lines with a gauge of 1 inch.

8. Must lines be straight in order to be parallel?

9. Draw two circles, one inside the other and both having the same centre,—*concentric circles,*—with radii $3\frac{1}{2}$ and 4 inches respectively. Describe the circumferences of these circles with respect to each other.

10. Why are the "parallels of latitude" so called?

Exercise 22.

1. Are all straight lines that do not intersect necessarily parallel? Study two sets of telephone wires, one running north and south and the other east and west.

2. Examine the edges of a box and point out the pairs of non-intersecting lines which are:

(*a*) parallel; (*b*) not parallel.

3. Consider a pair of each kind of lines separately. If these lines represented straight beams, could you lay a plank across one kind as well as across the other? Why?

4. Draw a pair of each kind of lines. Can you draw one kind as readily and accurately as the other, on a flat sheet of paper? Why?

A surface, such as that represented by a flat piece of paper, is called a *plane.* (Note spelling of *plain* and *plane.*)

5. What can you say of a pair of parallel straight lines that you cannot say of the other kind of non-intersecting straight lines?

Two parallel straight lines lie wholly in the same plane.

Exercise 23.

1. Draw two straight lines in as many different positions relative to each other as you can conceive, and lying in the same plane. Does the following classification cover all the cases?

Two Straight Lines

Intersecting (Forming an angle) — Non-intersecting (Parallel)

Intersecting: Perpendicular (Forming a right angle) — Oblique (Forming an oblique angle)

2. What are the distinguishing characteristics of perpendicular lines?

A statement of distinguishing characteristics is called a *definition.*

Exercise 24.

1. If the circumferences of two circles meet, how many points may they have in common without falling together, or *coinciding?* Illustrate by a drawing.

2. In how many points may a straight line meet, or intersect, the circumference of a circle?

3. How often can two straight lines intersect?

4. How many straight lines can you draw through a given point? Between two given points? How many points, then, does it take to locate a straight line definitely?

5. What must be true of a third point, if you want to draw a straight line through three points?

6. Can you suggest how to stake off a straight line, running from one end of a field to another? How does a surveyor "run" a straight line?

B. Formation of Polygons. Construction of Triangles.

Exercise 25.

1. What is the least number of lines required to form a figure closed on all sides?

2. Draw a closed figure with four sides; five sides. How many corners has the four-sided figure? The five-sided?

All closed figures have as many sides as angles and are named by lettering their vertices with capitals, starting from left to right, or opposite to the way that the hands of a clock move, i.e. *counter-clockwise.*

Polygon means *many corners* and is a term used in speaking of any closed figure having as many sides as angles.

3. A polygon of three sides is also called a triangle. Why?

4. Draw a triangle and name it.

5. Draw a polygon of six sides and name it.

Exercise 26.

1. Suppose a triangle *ABC* (Fig. 21) is drawn on the board and you wish to draw one equal to it. What parts will it be sufficient to measure? Write down how you would build up, or *construct*, your triangle, and state why you think that it will be an exact copy.

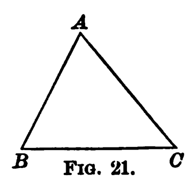
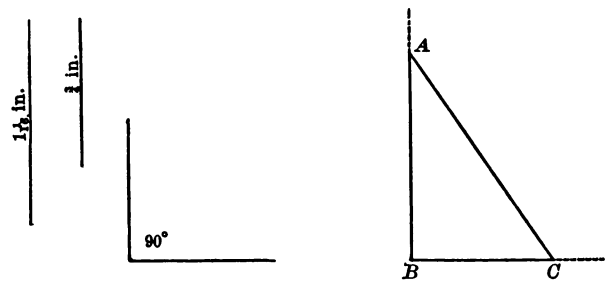


Fig. 21.

The pupils will discover incidentally that they need to measure only three of the parts to locate the three vertices of the triangle, and that they do not all need to measure the same three parts.

2. Construct a triangle ABC, for which the following measurements are given: $AB = 1\frac{1}{16}$ inches, $BC = \frac{3}{4}$ inch, and angle $B = 90°$. Describe the process and show how your drawing satisfies the given conditions.

SPECIMEN SOLUTION.



Construction. — Draw an angle equal to 90°, naming the vertex B. Make one of its sides equal to $1\frac{1}{16}$ inches, and call it BA. Then make the other side $\frac{3}{4}$ inch long and call it BC. Join points A and C. Then ABC is the required triangle.

Proof. — ABC is a triangle, having

$$\angle ABC = 90°,$$

$$AB = 1\tfrac{1}{16} \text{ inches},$$

$$BC = \tfrac{3}{4} \text{ inch},$$

which satisfies the given conditions.

Exercise 27.

Construct six triangles, ABC. Describe the process and show in each case how your drawing satisfies the given conditions. (The sign $\angle$ is used as an abbreviation for *angle.*) The measurements are:

	AB	BC	CA	$\angle A$	$\angle B$	$\angle C$
1.	$3\frac{7}{8}$ in.	$1\frac{1}{8}$ in.			85°	
2.		3 in.	3 in.			55°
3.			$2\frac{3}{8}$ in.	30°		115°
4.	4 in.			80°	40°	
5.	$3\frac{5}{8}$ in.	$1\frac{3}{4}$ in.	$2\frac{7}{8}$ in.			
6.		2.25 in.	3.75 in.		55°	

7. In a triangle ABC, $AB=3$ inches, $BC=2$ inches, and $\angle B = 60°$. Construct the triangle and measure the line CA and the angles A and C.

8. What are the lengths of BC and CA in problem 4?

9. In problem 2, how long is AB? How large is $\angle A$? $\angle B$?

10. Can a triangle be made, having for sides 2 inches, 1 inch, and 5 inches? $1\frac{1}{2}$ inches, $2\frac{1}{2}$ inches, and 4 inches? Why?

11. Two sides of a triangle are 2 inches and 5 inches. Between what limits must the third side lie?

Construct six triangles, *ABC*, from the following measurements:

	AB	*BC*	*CA*	$\angle A$	$\angle B$	$\angle C$
12.	45 mm.	108 mm.			35°	
13.		54 mm.	54 mm.			45°
14.		72 mm.			67½°	45°
15.	77 mm.			65°	25°	
16.	54 mm.	28 mm.	75 mm.			
17.	64 mm.	30 mm.				100°

The denominations used in this set of exercises belong to a system of measures known as the *Metric System*, which originated in France during the early part of the nineteenth century and has since been adopted by other nations. It is generally used in scientific work. The unit of length is the *metre*, a word meaning *measure*, as in gas-metre, water-metre, etc. It is a little longer than our yard. Figure 22 shows the decimetre divided into centimetres, one centimetre being divided into millimetres.

The abbreviation for kilometre is km.; for metre, m.; for decimetre, dm.; for centimetre, cm.; and for millimetre, mm.

Table:
1 km. = 1000 m.
1 m. = 10 dm.
1 dm. = 10 cm.
1 cm. = 10 mm.

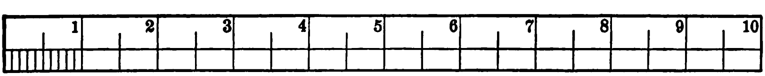
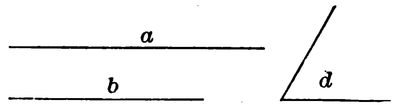


FIG. 22. — The decimetre divided into centimetres, one centimetre being divided into millimetres.

Exercise 28.

1. Draw any two lines and any angle. Form the triangle which contains these parts, the angle being included between the two lines. Call the lines a and b, and the angle d.



These letters are used in the algebraic sense, so that a may be any particular number of inches, b any other particular number of inches, and d any particular number of degrees.

Construct four triangles, ABC, from the following given parts. (See Exercise 26, page 26.)

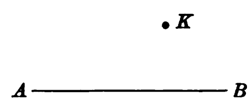
	AB	BC	CA	$\angle A$	$\angle B$	$\angle C$
2.	a	b			$e°$	
3.			c	$d°$		$g°$
4.		b	c			$g°$
5.	a	b	c			

6. Can you construct a triangle, no matter what lengths a, b, and c represent?

Exercise 29.

1. What is meant by the height, or *altitude*, of a building? Of a mountain? Show by a drawing. What, in your drawing, represents the *base?* What angle is formed by the altitude and a line in the "base" which meets it?

2. AB is a straight road and K is a kite directly over it. What line will represent the height, or altitude, of the kite?



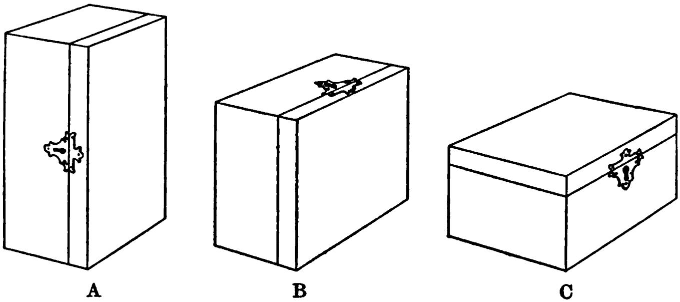


FIG. 23.

3. Figure 23 shows a box in three different positions, or resting on three different sides as bases. What line represents height in position A? In position B? In position C?

4. Triangle ABC (Fig. 24) may be said to stand on BC as a base. In that case, what is its altitude? If turned over so as to rest on AB as a base, what will be its altitude? What, if AC becomes the base?

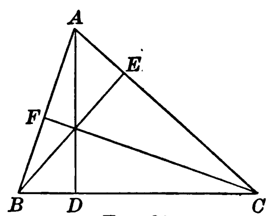
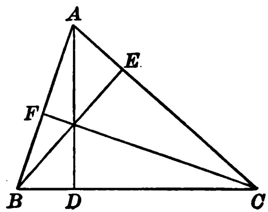


FIG. 24.

An *altitude of a triangle* may be described as the perpendicular drawn from a vertex to the opposite side considered as the base. There are three sides, each one of which may be made the base; hence, there are three possible altitudes.

Exercise 30.

1. Make any triangle, ABC, and then, with the aid of the T-square, draw its three altitudes, naming them AD, BE, and CF, respectively. Can you construct the triangle ABC, if you know the lengths of BC, AD, and BD? Show how.



2. Construct a triangle, ABC, when:

(a) $BE = 2\frac{7}{8}$ in., $CE = 3$ in., and $EA = 1\frac{1}{8}$ in.

(b) $AC = 60$ mm., $CE = 45$ mm., and $\angle C = 55°$.

(c) $BC = 3\frac{1}{2}$ in., $AD = 1\frac{3}{4}$ in., and $BD = 1$ in.

(d) $BD = 43$ mm., $AD = 54$ mm., and $DC = 24$ mm.

(e) $AF = 1\frac{1}{4}$ in., $FC = 2\frac{1}{4}$ in., and $\angle FCB = 40°$.

(f) $DC = 100$ mm., $AC = 111$ mm., and $\angle BAD = 30°$.

CHAPTER IV.

RELATIONS OF ANGLES IN PLANE FIGURES

Exercise 31.

1. A man holding a pocket compass directly in front of him walks east, and changes the direction of his motion first to north, then consecutively to northwest, west, and south, and finally again to east.

(*a*) What will he notice about the position of the needle with respect to the dial of the compass?

(*b*) What angles measure the consecutive changes of direction?

(*c*) What angle measures the total change?

2. If a peg was placed on the hand of a clock near the centre, what effect would the revolution of the hand have on it? Through what angle would it be turned?

3. Draw any four-sided figure, *ABCD*. Place a pencil on *AB* and slide it along *AB* until the rear end of the pencil reaches *B*. Then turn the pencil into the direction of *BC*, and continue on *BC* as on *AB*, turning at *C*, and so on, until the pencil is back again on *AB*.

Prove that the pencil turned completely around in moving around the figure.

4. A man starts from the corner of Garfield Park (Washington) at the intersection of East First Street and E Street (Fig. 25), and walks east along E Street, then north on East Third Street to Massachusetts Avenue, then north-

west on Massachusetts Avenue to West Third Street, then south on West Third Street to Canal, then along Canal to Garfield Park, and back to place of starting.

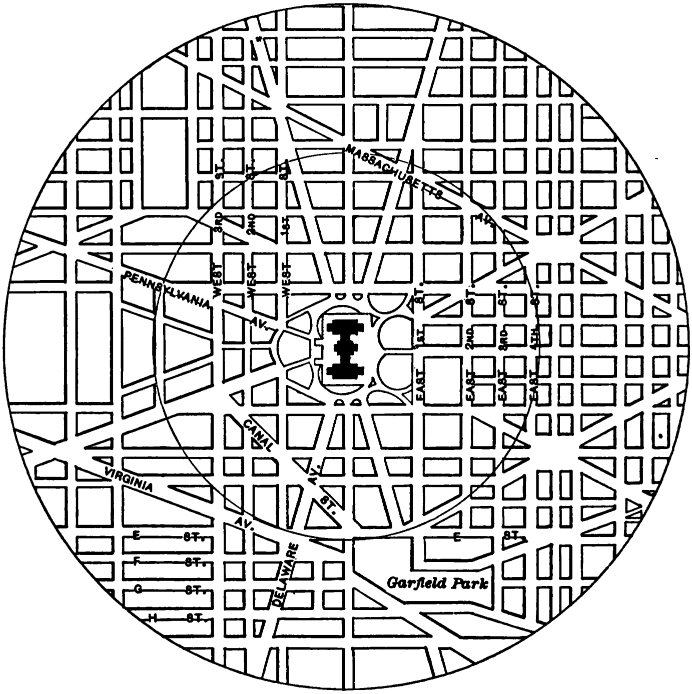


FIG. 25.

(*a*) How often has he changed his direction?

(*b*) Draw a diagram showing his course. What sort of a figure is it?

(*c*) How can you show in this diagram how much he turns from his course at each corner?

(*d*) What relation exists at each corner between this angle

showing change in course and the angle of the figure at the same vertex?

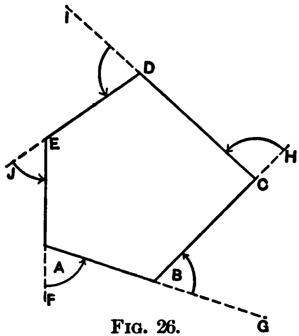


Fig. 26.

5. In Fig. 26, what is the relation between the two angles at A? At B? At C? Why?

Of the two angles at each of the vertices, the one on the outside of the polygon is called the *exterior angle*, to distinguish it from the one on the inside, or the *interior angle* of the polygon.

Exercise 32.

Draw any two five-sided polygons. Measure their angles (interior and exterior) and tabulate the results as follows:

Measurement at	Interior Angle	Exterior Angle
A		
B		
C		
D		
E		
Sum		

1. Do you notice anything peculiar in these results?

2. What is the sum of the exterior angles of each polygon? Do you see any reason why that should be so? Can you arrange the exterior angles of each polygon around a point or common vertex? Why?

3. Is it necessary to measure both the exterior and the interior angles at each vertex to make up the above table?

4. How can we find the sum of the interior angles of a polygon without measuring them?

Exercise 33.

If the sum of the interior angles of a polygon is equal to twice as many right angles as the figure has sides less four right angles, determine the sum of the interior angles of:

1. A six-sided polygon, or *hexagon.*

2. An eight-sided polygon, or *octagon.*

3. A ten-sided polygon, or *decagon.*

4. An n-sided polygon.

5. A four-sided polygon, or *quadrilateral.*

6. A triangle.

7. Take a triangular piece of paper. Clip off the corners and place them side by side with their vertices meeting in the same point. What does this prove about the sum of the angles of the triangle? Why?

8. Find the sum of the angles in a polygon of twenty-four sides.

9. In what polygon is the sum of the angles three times as great as in a pentagon?

Exercise 34.

A polygon with equal sides and equal angles is called *regular*.

1. What is the size of every exterior angle of a regular hexagon? What is, therefore, the size of each of its interior angles?

2. Determine in like manner the sizes of the interior angles of:

(*a*) A regular octagon.
(*b*) A regular decagon.
(*c*) A regular quadrilateral, or square.
(*d*) A regular triangle, or equilateral triangle.
(*e*) A regular n-sided figure.

Exercise 35.

1. Two angles of a triangle measure 36° and 65°, respectively. How many degrees does the third angle contain?

2. Two angles of a triangle are equal. How large is each, if the third angle is

(*a*) 36°? (*d*) 60°?
(*b*) 69°? (*e*) $m°$?
(*c*) 90°?

3. (*a*) In triangle ABC, $\angle C = 84°$. How large is the exterior angle at C? What is the sum of the other two interior angles? Write $\angle A + \angle B = \cdots$.

What is $\angle A + \angle B$, when $\angle C$ is

(*b*) 45°? (*d*) 72°?
(*c*) 60°? (*e*) $m°$?

4. What is $\angle B$, if $\angle C = 48°$ and the exterior angle at $A = 124°$? Make a statement regarding an exterior angle of a triangle.

5. How many obtuse angles can a triangle contain? How many right angles? (Try to draw a triangle containing two right angles.)

6. In a right triangle, what is the sum of the angles adjacent to the side opposite the right angle?

The side of a right triangle opposite the right angle is called *hypotenuse.*

7. ABC is a right triangle with the right angle at B and $\angle C = 30°$. If a line BN is drawn so as to make $\angle ABN = 60°$, what will be the size of $\angle ANB$?

8. Construct a triangle ABC, when $AC = 2$ inches, $BC = 3$ inches, and $\angle A + \angle B = 135°$

Exercise 36.

1. A triangular piece of land is enclosed by two lanes and a railroad track. The two lanes cross each other at an angle of $22\frac{1}{2}°$, and are equal in length. A telegraph line, 1000 yards long, enters the field at the crossing of one of the lanes with the railroad track and runs straight across it, passing over the other lane at right angles. Make a diagram of the land, representing the telegraph line, by a line $\frac{9}{16}$ of an inch long.

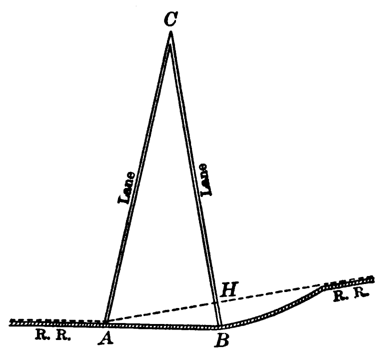


FIG. 27.

SPECIMEN SOLUTION.

Make a rough drawing representing the conditions stated in the problem. With this drawing, which we will call a *trial figure*, it will be easy for you to discover the solution by the following method.

Study of trial figure. — Let ABC be the triangle and AH represent the telegraph line. (See Fig. 27.) Into what two parts does AH divide the triangle ABC? How large is angle CAH? Is it possible to construct triangle AHC and then build on it the rest to complete the whole?

Construction. — Draw a line $\frac{9}{16}$ of an inch long. Call it AH. At A, and with AH as one side, make an angle equal to $67\frac{1}{2}°$. Likewise, at H make an angle of $90°$. Let the unnamed, or *free*, sides of these angles meet at C. Make CB equal to CA. Join A and B. Then ABC is the required triangle.

Proof. — $CA = CB$.

$AH = \frac{9}{16}$ of an inch.

AH meets BC at right angles.

$\angle C = 180° - 90° - 67\frac{1}{2}° = 22\frac{1}{2}°$.

Therefore triangle ABC satisfies the conditions of the problem.

2. Draw the diagram, if the telegraph line

(*a*) bisects BC.
(*b*) bisects $\angle A$.

3. Draw a triangle ABC (Fig. 24), when

(*a*) $AC = 3$ in., $AD = 2$ in., and $BC = 4$ in.
(*b*) $AD = 2$ in., $\angle C = 40°$, and $\angle A = 75°$.
(*c*) $AB = 3$ in., $AD = 2$ in., and $AC = 4$ in.

Exercise 37.

1. Make a triangle and name it ABC. Draw from A:

(*a*) The altitude to BC.

(*b*) The line bisecting the angle A.

The portion of this line between A and the opposite side BC is called an *angle-bisector*. See AN in Figs. 28 and 29.

(*c*) The line joining A with the middle point of BC.

This line is called the *median* from A.

2. Is it possible to draw separate lines for the altitude, median, and angle-bisector from each of the vertices of every triangle? Try in different triangles.

(*a*) What do you find in a triangle having all its sides unequal? In one having all its sides unequal, and one angle equal to 90°?

(*b*) What do you find in a triangle having two of its sides equal? In one having two of its sides equal, and one angle equal to 90°?

(*c*) What do you find in a triangle having all its sides equal?

As these lines may lie very close together, it will be convenient to name each one so as to indicate:

(i) The kind of line it is, whether *altitude*, *angle-bisector*, or *median*.

(ii) Its position, whether drawn from A, B, or C.

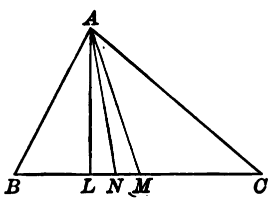


FIG. 28.

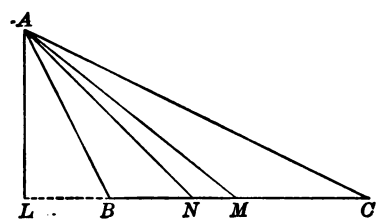


FIG. 29.

In Figs. 28 and 29 the letters L, M, and N are used to name the points of intersection of these lines with the side opposite to the vertex from which they are drawn. These letters are the first consonants in the words *aLtitude*, *Median*, and *aNgle-bisector*, respectively, and may thus — in a way — suggest the words themselves.

3. Let L, L' (L-prime), and L'' (L-second) be the points where the altitudes from A, B, and C, respectively, meet the opposite sides.

Then AL is the altitude from A to BC,
BL' is the altitude from B to AC,
and CL'' is the altitude from C to AB.

Likewise, let M, M', and M'' be the points where the medians from A, B, and C, respectively, cut the opposite sides; and N, N', and N'' be the points where the lines bisecting the angles A, B, and C, respectively, meet the opposite sides.

Then BM' represents the median from B to AC;
$AN \doteq$ the bisector of $\angle A$, etc.

What does each of the following represent:

(*a*) BN', CM'', CN'', AM?
(*b*) $\angle LAN$, $\angle L'BM'$?

4. Draw a triangle ABC and locate:

(*a*) AM, BN', and CL''.
(*b*) AN, BL', and CM''.
(*c*) AL, BM', and CN''.
(*d*) $\angle MAC$, $\angle BM'A$, and LAN.

5. What relation exists between $\angle A$ and angles MAC and BAM?

6. What relation exists between $\angle A$ and $\angle ACL''$?

Exercise 38.

Construct a triangle, ABC, from the following given parts, drawing a trial figure to determine the process of construction:

A.

1.	$BC = 3\frac{7}{8}$ in.	$AC = 3\frac{1}{8}$ in.	$BM' = 3\frac{1}{4}$ in.
2.	$BC = 3$ in.	$\angle C = 25°$	$BM' = 2$ in.
3.	$BC = 4$ in.	$AM = 2\frac{1}{2}$ in.	$\angle CAM = 36°$
4.	$BC = 5$ in.	$\angle B = 55°$	$BN' = 4\frac{1}{2}$ in.
5.	$AN = 4\frac{1}{4}$ in.	$\angle A = 36°$	$\angle B = 120°$
6.	$BC = 2\frac{3}{8}$ in.	$\angle AMC = 100°$	$\angle MAC = 40°$
7.	$BC = a$	$\angle B = q°$	$AM = m$
8.	$BC = 3$ in.	$CL'' = 2\frac{1}{2}$ in.	$CM'' = 3\frac{1}{2}$ in.
9.	$BC = a$	$AL = l$	$\angle C = r°$
10.	$\angle A = p°$	$AL = l$	$AN = n$

B.

11.	$AC = 111$ mm.	$BC = 72$ mm.	$AM = 78$ mm.
12.	$AB = 69$ mm.	$BC = 76$ mm.	$CM'' = 63$ mm.
13.	$AC = 64$ mm.	$\angle C = 57°$	$\angle CAM = 36°$
14.	$AB = 54$ mm.	$AN = 68$ mm.	$\angle A = 40°$
15.	$AC = 70$ mm.	$AL = 50$ mm.	$AM = 60$ mm.
16.	$AB = 54$ mm.	$\angle A = 35°$	$\angle BM'A = 70°$
17.	$AB = c$	$\angle A = p°$	$CM'' = m$
18.	$CN'' = 62$ mm.	$\angle A = 70°$	$\angle B = 55°$
19.	$\angle B = q°$	$AL = l$	$\angle C = r°$
20.	$AB = c$	$AL = l$	$AN = n$

Exercise 39.

1. Collect as many triangles, different in shape, as you can. Compare and contrast each with all the others. Group those which have points in common.

2. Classify the triangles according to:

(*a*) Relation of sides;
(*b*) Relation of angles.

(*a*)

Triangles

- **All sides equal** (*Equilateral*)
- **Two sides equal** (*Isosceles*)
  - **Largest angle** *acute*
  - **Largest angle** *right*
  - **Largest angle** *obtuse*
- **No sides equal** (*Scalene*)
  - **Largest angle** *acute*
  - **Largest angle** *right*
  - **Largest angle** *obtuse*

(*b*)

Triangles

- **Largest angle** *obtuse*
  - **Others equal**
  - **Others unequal**
- **Largest angle** *right*
  - **Others equal**
  - **Others unequal**
- **Largest angle** *acute*
  - **All equal** (*Equiangular*)
  - **Two equal**
  - **All unequal**

3. Show how the triangles are named according to their main characteristics. Define a right isosceles triangle.

Exercise 40.

1. If one of the base angles of an isosceles triangle is 36°, what is the angle at the top?

2. How large is each one of the acute angles in an isosceles right triangle?

3. What angle is formed by the bisectors of any two angles in an equilateral triangle?

4. Draw a scalene triangle ABC. If $\angle A = 50°$, $\angle B = 60°$, and $\angle C = 70°$, find the angles formed by:

(*a*) BN' and CN''; (*b*) AL and CL''; (*c*) AL and AN.

5. The bisector of a base angle in an isosceles triangle makes, with the opposite side, an angle equal to 57°. How large are the angles of the triangle?

6. How large are the angles of an isosceles triangle whose altitude is equal to half its base?

Exercise 41.

1. Make an isosceles triangle, ABC, and with the vertex A as centre and one of the equal sides, AB, as radius, draw a circle.

(*a*) Why does the circumference of this circle pass through C?

(*b*) If you wish to make a triangle, ACD, equal to ABC and adjacent to it, what will be the quickest way to do it?

HINT. Draw the chord CD equal to BC.

2. Tom wants to draw an angle equal to a given angle BAC, and having A' for its vertex and $A'B'$ for one of its sides. He places one end of his compasses on A and describes an arc cutting the sides of angle BAC in X and Y. Then he constructs an equal arc about A' as vertex, cutting $A'B'$ at a point which he calls X'. With XY as a radius, he then describes an arc about X' as centre, cutting the last arc at Y'.

Repeat this construction on paper and show by folding or measuring that $\angle X'A'Y' = \angle BAC$.

This is copying an angle without the aid of the protractor, using only the ruler and the compasses.

CHAPTER V.

ANGLES ON PARALLEL LINES—THE PARALLELOGRAM.

Exercise 42.

1. Construct triangle ABC, when $AB = 1$ inch, $\angle ABC = 90°$, $\angle BAC = 30°$ (Fig. 30).

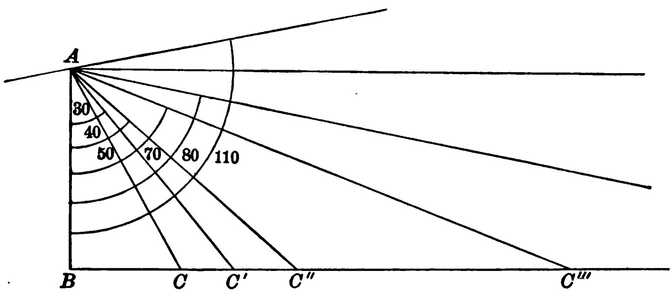


FIG. 30.

(*a*) How large is $\angle C$?

(*b*) How large would $\angle C$ be, if $\angle A$ were made equal to 40°, 50°, 80°? Locate point C for each case.

(*c*) How large must $\angle A$ be made, in order that $\angle C$ may be 5°, 1°? Try to locate point C.

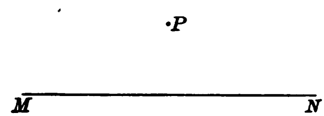
(*d*) What value of $\angle A$ will make it impossible for the $\angle C$ to form? Why?

(*e*) What happens if $\angle A$ grows to be 110°? In this case, can the free sides of the angles A and B be made to meet at all?

2. Answer (*b*), (*c*), and (*d*), when $\angle B = 65°, 100°$.

3. For what value of *A* will the free sides of angles *A* and *B* not meet at all, if $\angle B = 60°, 45°, 90°, 135°$? Show by drawings. Why may the free sides of the angles *A* and *B* in these cases be called parallel? (See Exercise 21.)

4. Does this suggest to you a way of drawing a line through the vertex of a triangle parallel to the base? Then draw a parallel to a given straight line *MN*, through a point *P* outside of it? How many such parallel lines can you draw through *P*?



5. Can you place two points so that only one line can be drawn through them parallel to a given straight line?

6. How many sets of straight lines parallel to each other can you draw through two points?

Exercise 43.

1. Draw two parallel lines and any line, not a perpendicular, cutting both of them. Examine the angles that are formed. How many are there? How are they placed

(*a*) With reference to the cutting line?
(*b*) With reference to the parallels?

2. What is the sum of the two angles on the same side of the cutting line and within the parallels? Why? How many pairs of such angles are there?

3. What is the sum of the two angles on the same side of the cutting line and outside the parallels? Why?

4. What angles that are neither adjacent nor vertical, are equal? Why?

5. Chart II pictures four mechanical ways of obtaining parallel lines. Give reasons why the lines obtained in each case are parallel.

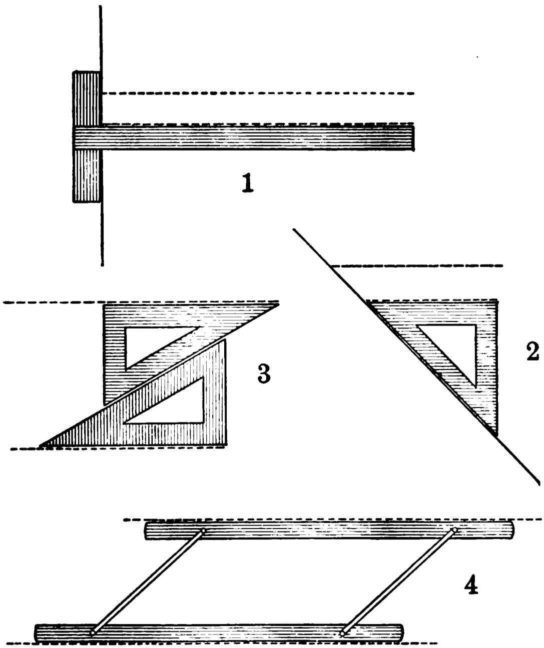


CHART II.

Exercise 44.

1. Draw two pairs of parallels, one crossing the other. Observe the four-sided figure thus formed. It is a diagram with parallel sides, or *parallelogram.* A parallelogram is thus a four-sided figure, the opposite sides of which are parallel. What relations exist among its angles? Why?

2. How large are the other angles, if one is

(a) 36°? (b) 115°? (c) 90°? (d) $x°$?

3. How many altitudes do you distinguish in the parallelogram? Why not four?

4. Measure the sides of your parallelogram. State which are equal.

Why is the quadrilateral whose vertices are the pivots of the parallel ruler (Chart II) always a parallelogram, whether the ruler be opened or closed?

5. Draw as many different parallelograms as you can. Compare and contrast each with all the others. Group those which have points in common.

Parallelograms with

- **Right angles and**
  - **Equal adjacent sides** (*Square*)
  - **Unequal adjacent sides** (*Rectangle*)
- **Oblique angles and**
  - **Equal adjacent sides** (*Rhombus*)
  - **Unequal adjacent sides** (*Rhomboid*)

6. What is the difference between

(a) A square and a rectangle?
(b) A rhombus and a rhomboid?
(c) A rhombus and a square?

7. In what respect do the square and the rhombus agree?

8. In what do the rectangle and the square agree?

Exercise 45.

1. Draw any angle, *MON*, and across its sides pass two parallel lines KK' and LL'. The resulting quadrilateral, *ABCD*, is called a *trapezoid* (Fig. 31).

(*a*) How does it differ from a parallelogram?

(*b*) How could you obtain a trapezoid from a parallelogram? From a triangle?

(*c*) Draw all its altitudes. Which one is commonly taken as altitude? Why?

(*d*) What relations exist among its angles?

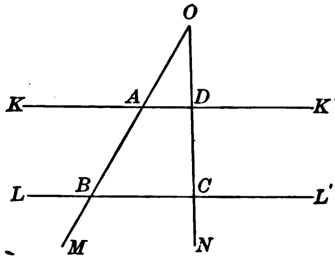


FIG. 31.

2. What must be the conditions in a certain trapezoid, *ABCD*, if the sides *BA* and *CD*, prolonged until they meet, give an isosceles triangle? What special name would be appropriate for such a trapezoid?

3. Draw any trapezoid, *MNOP* (*MN* parallel to *OP*), and cut out of paper another one equal to it, naming it *M'N'O'P'*. (Read *M*-prime *N*-prime *O*-prime *P*-prime.) Place this second figure by the side of the first letting *O'* fall on *N*, and *N'* on *O'* (Fig. 32).

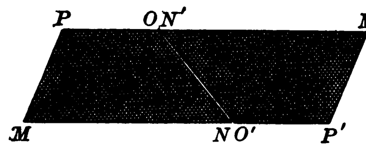


FIG. 32.

(*a*) What sort of a figure do the two make? Prove your statement.

(*b*) What relation, in size, is *MNOP* to *MP'M'P*?

4. Define: Square, Rhombus, Rectangle, Rhomboid, Trapezoid. What common characteristic have all five of these figures?

CHAPTER VI.

SYMMETRY.

A. Fundamental Problems of Construction.

Exercise 46.*

1. If you want to make a pen-wiper, as shown in Fig. 33,

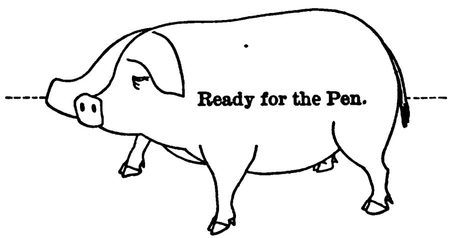


Fig. 33.

how will you cut out the outline of the pig (Fig. 34)? Why?

2. If a seamstress wants to cut out the back of a dress, she places the pattern of half the back on the lengthwise fold of the goods. Why does she do that?

3. Name, or draw, some figures which may be made that way.

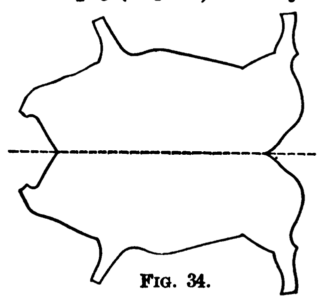


Fig. 34.

* To the Teacher. This exercise is given for the purpose of developing the idea of *symmetry*, so-called *axial symmetry*. It is merely suggestive and should be amplified, if necessary. Teach the terms *symmetrical* and *axis of symmetry*.

4. Examine the leaves in Chart III. What main difference do you note in their structure?

5. How would you describe the structure of the two halves of the moth (Fig. 35), with reference to the straight line drawn lengthwise through its body?

6. What word do you know which will describe likeness in structure between two parts of an object or a figure?

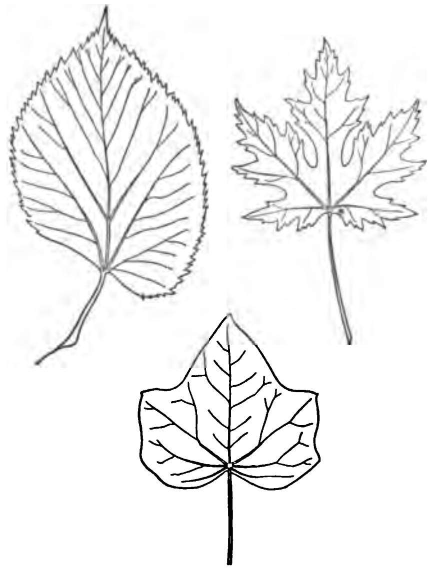


CHART III.

Figures are said to be *symmetrical* with reference to a line, if the two parts on the opposite sides of this line fit exactly when folded one over the other.

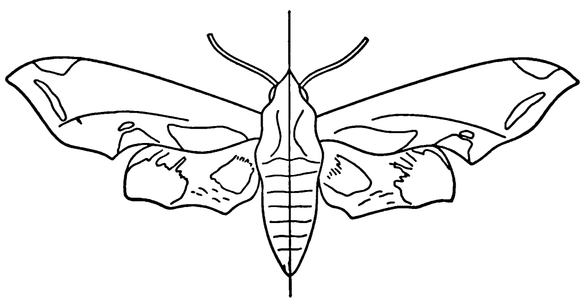


FIG. 35.

Exercise 47.

1. Take a paper circle and draw one of its diameters, dividing the circle into two parts. Place a number of ink-spots on one part and fold the paper over on the diameter before the ink is dry, so as to make an impression on the other part. Connect each spot, or point, by a line with its corresponding blot, or *counterpoint*.

(*a*) What relation exists:

(i) Between the parts of the lines connecting the points?

(ii) Between the angles formed at the intersection of each of these lines with the axis of symmetry?

Test with the square or the protractor.

(*b*) How, then, can you find the counterpoint to each point without folding as at first?

Use a square to draw the perpendicular to the axis of symmetry from the point whose counterpoint you wish to locate. Then prolong this line its own length through the axis of symmetry. The end of this line will be the desired point.

2. Fold a piece of paper, run a pin-point through the two parts of the paper, and then open. Call the crease MN, the two pinholes A and B, and draw straight lines from A and B to any point P in MN, and not in a straight line with A and B.

(*a*) What will happen if the paper be folded again as at first:

(i) To the lines? (ii) To the angles?

(*b*) Given a straight line MN as an axis of symmetry and a point A outside the line, what method of constructing B as counterpoint of A does this suggest?

Use the compasses to make $\angle BPN = \angle APN$ and $BN = AN$. (See Exercise 41, page 44.)

Exercise 48.

1. Draw a straight line, MN (Fig. 36), and from any point, P, in it draw another straight line, PQ, making an acute angle with MN. With reference to MN as an axis of symmetry, construct the counterpoint to Q:

(*a*) With the aid of a T-square or the protractor;

(*b*) Without either, using the compasses instead.

2. Construct the symmetric figures suggested in Chart IV, using the ruler and the compasses.

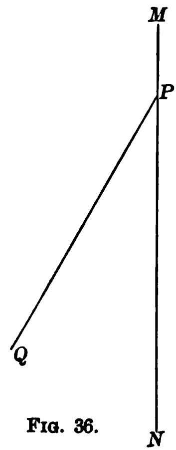


FIG. 36.

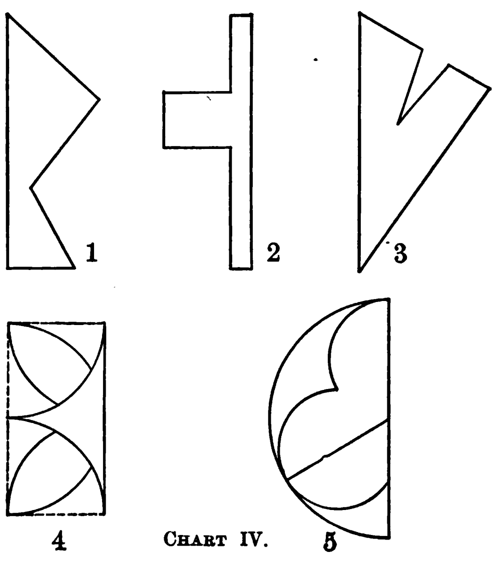


CHART IV.

3. Draw an isosceles triangle, ABC, and bisect A, the angle at the vertex, the latter being 40°. State what will happen if ABD is folded over on AD as an axis (Fig. 37).

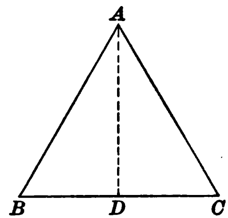


FIG. 37.

(a) In what direction will BA fall? Why?

(b) How large is $\angle CDB$? Then, how large is $\angle CDA$? $\angle BDA$?

(c) What relation exists between BD and DC? $\angle B$ and $\angle C$?

(d) What is the value of each of the angles in this figure? What, if $\angle A = 50°$? 90°? 60°?

(e) What different functions has AD in this isosceles triangle?

Exercise 49.

Given triangle ABC. With BC as an axis of symmetry, draw the counterpart of ABC. Draw AA', and join A and A' with E, F, and G, any three points in BC (Fig. 38).

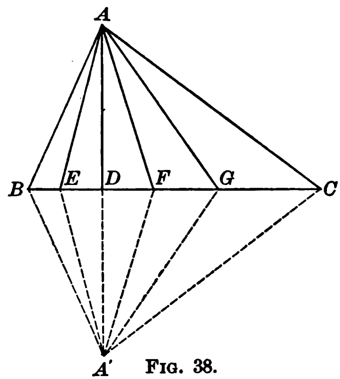


FIG. 38.

(a) Where will ABC fall when folded over on BC? Where will the line AE fall? AF? AG? What triangles will coincide? What lines are equal? What angles are equal?

(b) What kinds of triangles are ABA', AEA', etc.? What different functions has BC in each of these triangles? How many more isosceles triangles can you draw, having AA' for a base? Where must their vertices lie? Why?

Exercise 50.

1. Given the triangle ABC. With AB as axis of symmetry, draw the counterpart of ABC, using only the ungraduated ruler and the compasses. Then join C and C' and name the point where this line cuts AB, D. How large is $\angle ADC$? What function has AD in the triangle ABC?

2. Given a line, AB, and a point, P, outside of it, to drop a perpendicular to AB from P. Use only the ungraduated ruler and the compasses. How many different ways of constructing this perpendicular do you know now?

3. Construct a triangle whose sides measure 4, 3, and 2 inches respectively. Draw its altitudes, using only the ruler and the compasses.

4. Construct a triangle, ABC, with $AB = 4$ inches, $\angle A = 120°$, and $\angle B = 30°$. Draw its altitudes, using only the ruler and the compasses.

Exercise 51.

1. Given a line, AA', to construct a line, BC, such that every point of it may be made the vertex of an isosceles triangle with the given line, AA', as a base. What other functions has line BC?

2. Bisect a given straight line, using only the ungraduated ruler and the compasses.

3. A and B are two points 3 inches apart. With these points as centres, draw two circles, each having a radius of 2 inches. Connect A and B, and draw a line joining the points in which the circumferences intersect. How does this line divide AB? Why?

4. Given a line, AB, to find a line which will be perpendicular to it at its middle point.

5. Make any triangle and draw its medians, using only the ungraduated ruler and the compasses.

6. Given a line, AB, and a point, P, in it, to find a line which will be perpendicular to AB at the point P.

7. Erect a perpendicular to a line at one of its extremities.

8. Given two points, P and P', find the axis of symmetry with reference to which these points would be counterpoints. Construct without using the line PP'.

Exercise 52.

1. Draw an isosceles triangle and bisect the angle at the vertex:

(*a*) With the aid of the T-square or the protractor.
(*b*) Using only the ruler and the compasses.

2. Bisect a given angle. Use only the ruler and the compasses.

3. Without the use of the protractor, using only the ruler and compasses, construct the following angles:

(*a*) $45°$;	(*d*) $30°$;	(*g*) $75°$;
(*b*) $22\frac{1}{2}°$;	(*e*) $15°$;	(*h*) $105°$.
(*c*) $67\frac{1}{2}°$;	(*f*) $37\frac{1}{2}°$;	

4. Make any triangle and draw its angle-bisectors, using only the ruler and the compasses.

Exercise 53.

1. Draw a circle with an angle of 60° at its centre. Then draw the chord which *subtends* the angle, — that is, spans it or joins the ends of its arms, — and drop a line perpendicular to it from the centre. Show that the perpendicular line bisects the chord and the arc subtended by it.

2. Why does every diameter perpendicular to a chord bisect the chord and also the arc subtended by it?

3. Given a circular piece of paper. How can you find its centre without folding?

B. Symmetry of Regular Polygons.

Exercise 54.

1. Draw an isosceles triangle, ABC, with BC as the base. Construct the medians AM and BM', and call their point of intersection P (Fig. 39).

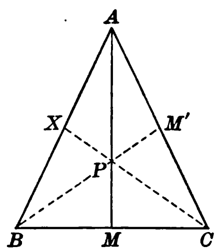


FIG. 39.

(*a*) What will be the position of BP on AMC, if ABM is folded over on AM as an axis? Of PM' on ABM?

(*b*) What relation exists between the angles BPX and CPM'? Why?

(*c*) What kind of an angle is BPM'? (See page 16.) $\angle CPX$?

(*d*) What relation exists between BX and XA? Why?

(*e*) What kind of a line in triangle ABC is the line CPX? What does this prove about the intersection of the three medians, AM, BM', and CX?

(*f*) In what three ways can you construct X, the counterpoint of M'?

2. Show, in the same way, that the three altitudes of triangle ABC (Fig. 39) also intersect in a common point.

3. Show, in the same way, that the three lines bisecting the angles of triangle ABC (Fig. 39) pass through a common point.

Exercise 55.

1. Construct an isosceles triangle, ABC, and from each one of its vertices draw the altitude, the median, and the bisector of the angle (Fig. 40). Do you obtain three lines from each vertex? Why do (i) the medians, (ii) the angle-bisectors, and (iii) the altitudes, from A and B, intersect in points on the line CD? Let P, Q, and R represent the points of intersection of the medians, angle-bisectors, and altitudes respectively.

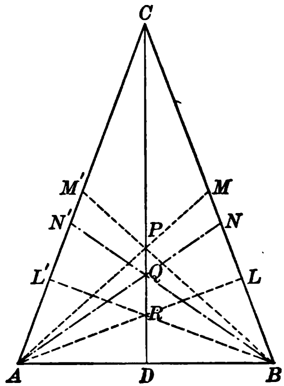


FIG. 40.

2. Draw other isosceles triangles, construct the altitudes, medians, and angle-bisectors as in Fig. 40, and examine the positions, of P, Q, and R with reference to each other.

(*a*) What is the position of R, when $\angle C = 1$ rt. $\angle$ (1 right angle)?

(*b*) What is the position of R, when $\angle C$ is obtuse?

(*c*) When will P, Q, and R coincide?

Exercise 56.

Draw an equilateral triangle, ABC. Why may it be regarded as symmetrical with reference to three different axes? (Fig. 41.)

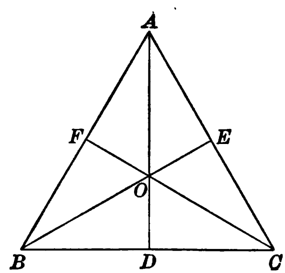


FIG. 41.

(*a*) What three functions has each axis?

(*b*) Why do these axes meet in one point O?

(*c*) Examine the angles formed by the three axes at O? Why are they equal?

Exercise 57.

1. (*a*) Is the square a symmetrical figure? Why? (*b*) The rectangle? (*c*) The rhombus? (*d*) The rhomboid? (*e*) The isosceles trapezoid? (*f*) The oblique trapezoid? (*g*) The regular pentagon? (*h*) The regular hexagon? (*i*) The regular octagon? (*j*) The circle?

2. How many axes of symmetry has each one of these figures?

3. Of the quadrilaterals, which has the greatest number of axes of symmetry? How do you account for this?

4. In the cases where you find two axes of symmetry, why are the angles which the axes form at their point of intersection equal?

The point at which the axes of symmetry meet is called the *centre of symmetry.*

Exercise 58.

1. What may be observed about the angles at the intersection of the axes of symmetry in regular polygons? (See definition on page 37.) In the equilateral triangle, Fig. 41, what is the relation of the angles AOB, BOC, and COA? How large is each? Make use of this knowledge by inscribing an equilateral triangle in a circle.

Angles like AOB, BOC, and COA in Fig. 41, at the centre of symmetry and each subtended by a side of the polygon, are called *angles at the centre.* Each regular polygon has as many of them as it has sides.

2. How large is the angle at the centre of a square? Of a regular pentagon? Of a regular hexagon? Of a regular octagon? Of a regular decagon?

3. In a given circle inscribe

(*a*) A square. (*c*) A regular pentagon.
(*b*) A regular octagon. (*d*) A regular decagon.

4. In a given circle, inscribe a regular hexagon.

(*a*) What kind of triangles do you observe grouped around the centre?

(*b*) What relation, then, exists between the sides of your hexagon and the radius of the circle?

5. Construct a regular hexagon with a side equal to a given length.

Exercise 59.

1. Obtain a regular hexagon by folding as follows:

(*a*) Take a square piece of paper (*1*, Chart V), and fold it in halves (*2*).

(*b*) Then fold in quarters (*3*) and eighths (*4*), obtaining creases *OA* and *BC*.

(*c*) Fold *OD* over, so that *D* touches the crease *BC* (*5*).

(*d*) Bring *OEG* over *OD*, letting *OE* fall on *OH* (*6*).

(*e*) Cut straight from *D* to *E* (*7*) and open. The result is a regular hexagon (*8*).

2. (*a*) What is the relation between *OD* and *OE*? *OE* and *DE* (*5*)?

(*b*) What kind of a triangle is *DOE* in *5*? In *7*?

(*c*) Why does *OE* fall on *OH*?

(*d*) Why is *8* a *regular* hexagon?

3. What kind of a figure will you get, if you cut from *D* (*6*) to a point on *OE*, one-fourth the length of *OE* from *O*?

4. Make a regular dodecagon (12 sides) by folding and cutting.

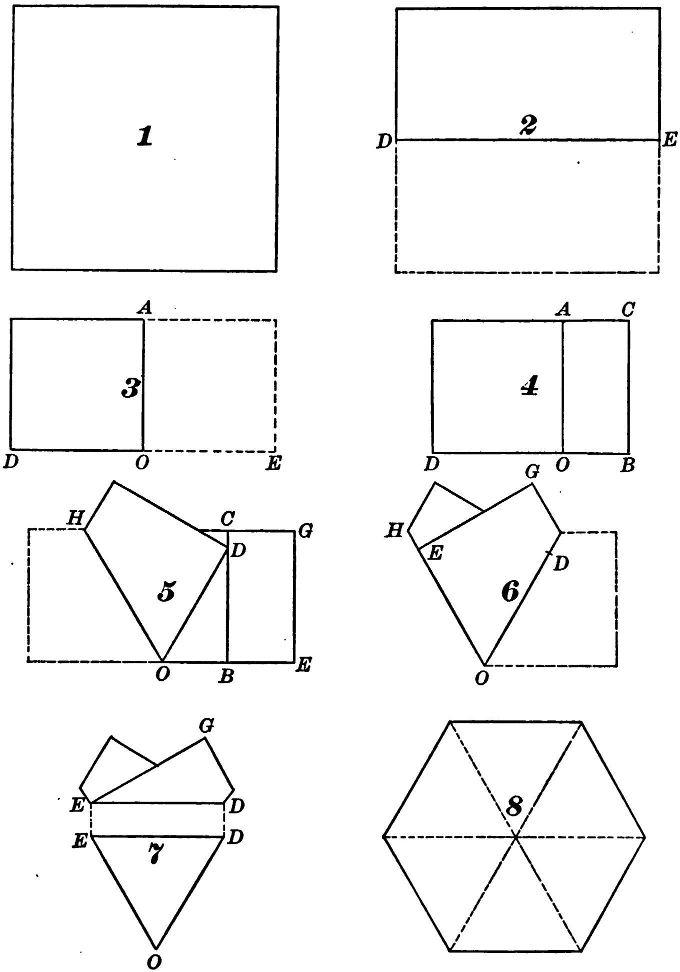


CHART V.

Exercise 60.

1. Make a regular pentagon by folding as follows:

(*a*) Take a piece of paper $4\frac{3}{4}$ inches by 5 inches (*1*, Chart VI).

(*b*) Fold lengthwise in halves (*2*) and mark *O*, the middle point of the crease *AB* (*3*).

(*c*) Turn up the corner *B* and place it so that *B* falls on *DC* (*4*).

(*d*) Proceed with *A* as with *B*, only turning it under (*5*).

(*e*) Fold the part *DLOB* in halves, laying *LO* beside *OB*. Measure *OM* equal to *OA* (*6*).

(*f*) Cut along *AM* and open (*7*).

2. (*a*) Is *8* a regular pentagon? How would you assure yourself that it is? (For definition of regular polygon, see Exercise 34, page 37.)

(*b*) How large is $\angle AOM$ in *7*?

3. Make a regular decagon by folding and cutting.

Exercise 61.

1. Fold a piece of paper twice at right angles and cut off the folded corner, making a rhombus when the part cut off is opened out. Can you cut out a rhombus having two angles of 60° each?

2. Can you so cut a piece of paper, folded twice at right angles, that the part cut off will be a square?

3. Fold a square piece of paper to obtain a regular octagon. How do you determine the line along which you cut? What happens, if you should cut at an angle of about 30° to this line?

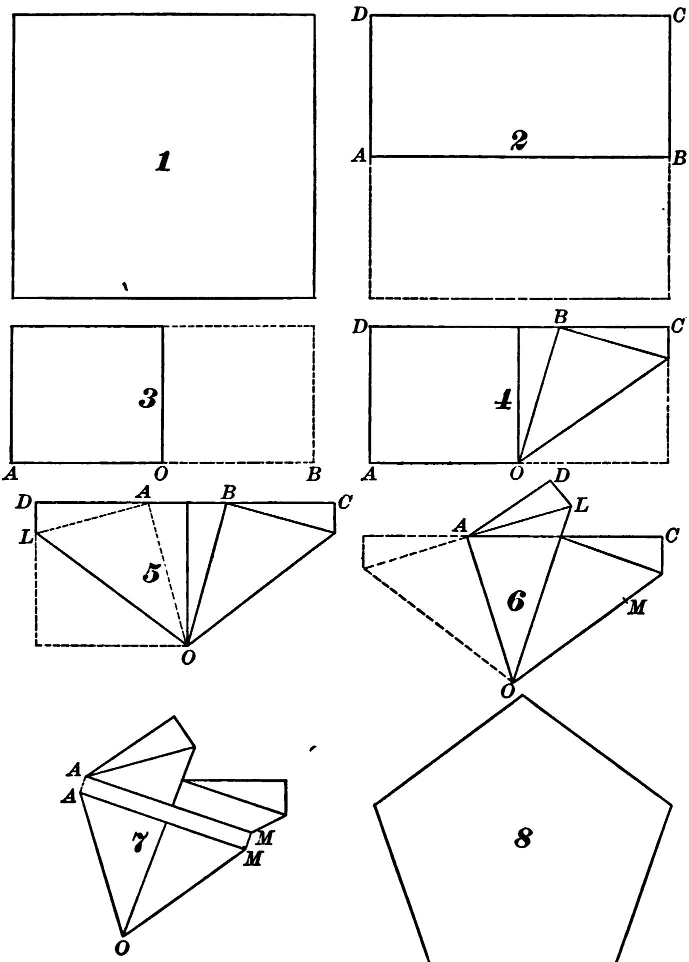


CHART VI.

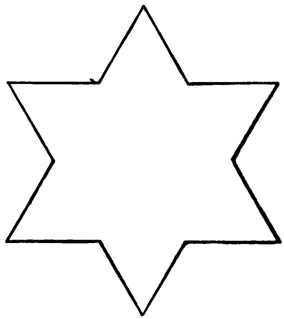


FIG. 42.

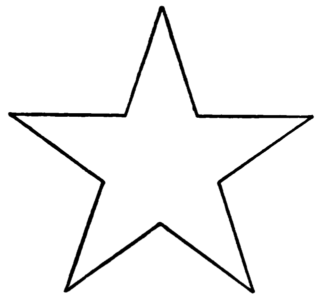


FIG. 43.

4. Give directions for cutting with one clip of the scissors:

(*a*) A six-pointed star (Fig. 42).
(*b*) A five-pointed star (Fig. 43).*
(*c*) A four-pointed star.

---

* "According to well authenticated tradition, George Washington, Robert Morris, and John Ross, in June, 1777, waited upon Ross's niece, Mrs. Elizabeth Ross, a noted needle-woman of Philadelphia, living at what was then 239 Arch Street, to procure from her a new flag, according to a design authorized by act of Congress. Congress had resolved that 'the flag of the United States be thirteen stripes, alternate red and white; that the Union be thirteen stars, white in a blue field, representing a new constellation.' Washington drew a rough draught, which he submitted to Mrs. Ross.

"'Is there anything about the flag that you can suggest, Mrs. Ross?'

"'Your excellency,' said Betsy, 'I see your stripes are white at top and bottom. Do you not think red better to begin and end with?'

"'Possibly,' said Washington, musingly.

"'And your stars are six-pointed. Would not a five-pointed star be better?'

"'It makes little difference,' was the answer. 'I conceived the six-pointed star easier to form. Is that not so?'

"'No, your excellency. See.'

"And with this, she caught up a piece of paper, folded it and then cut it, succeeding in less than thirty seconds to form a perfectly proportioned five-pointed star." — From a clipping of the *Chicago Record-Herald.*

Exercise 62.

1. Draw an equilateral triangle, ABC, and its three axes of symmetry, AD, BE, and CF (Fig. 41). What three ways have you of constructing these axes? Why?

2. Let the point in which the axes intersect, the centre of symmetry, be called O. Then, what can you say of

(*a*) OD, OE, and OF?
(*b*) OA, OB, and OC?

3. If you should draw a circle about O with OA as a radius, what other points would it include? What other points would it include if drawn with OD as a radius?

In one case, the sides of the triangle become chords and the circle is said to be *circumscribed*. In the other, the sides of the triangle merely touch the circle and the circle is said to be *inscribed*. When lines *touch* a circle without cutting it, they are called *tangents*.

Exercise 63.

1. In what respects do the axes of the square differ from those of the equilateral triangle

(*a*) As to length?
(*b*) As to function?

How is it in the regular pentagon, hexagon, etc.?

2. What would be the position of a circle about the centre of symmetry of a square, regular pentagon, etc., drawn with a radius equal to the latter's distance

(*a*) From the vertices?
(*b*) From the sides?

3. Draw the inscribed and circumscribed circles of:

(*a*) An equilateral triangle.
(*b*) A square.
(*c*) A regular hexagon.
(*d*) A regular octagon.

CHAPTER VII.

EQUALITY OF TRIANGLES — CONSTRUCTION OF TRIANGLES, PARALLELOGRAMS, AND TRAPEZOIDS.

Exercise 64.

1. Construct different* triangles, each having an angle equal to 60°. What relation exists among these triangles?

2. Construct different* triangles, each having one angle equal to 60° and another equal to 45°. What relation exists among these triangles? Is there as much difference between them as there is between those of problem 1?

The latter are different in size, but equal in form; whereas the former are different both in size and form.

3. Construct triangles,* each having one side equal to 2 inches, one angle equal to 40° and another equal to 72°. What relation exists among these triangles? How can you test it?

Figures that are *equal in size and form* may be made to fit when placed one upon the other.

4. What will happen, if two of the triangles are placed side by side, with a pair of equal sides falling together, and are then folded one over on the other?

---

* To the Teacher. When teaching a number of pupils, let each make only one.

Exercise 65.

1. If two figures can be placed adjacent to each other so as to form a symmetrical figure, what must be their relation to each other, in size and form? Why?

2. What is it sufficient to know about two triangles to ascertain whether they will form a symmetrical quadrilateral, if placed in that way? Supposing you want to form the quadrilateral $ABCA'$ (Fig. 38, page 54) from the triangles ABC and $A'B'C'$:

(*a*) What parts must necessarily be equal?

(*b*) Equality of how many more will be sufficient to make $ABCA'$ symmetrical with respect to BC? What parts may these be?

3. Given two triangles, with two sides and the included angle of one equal respectively to two sides and the included angle of the other. Can you place them together so as to form a symmetrical quadrilateral? Can you, then, place one upon the other so they will fit? What do you conclude with reference to the two triangles? Make a statement about this.

4. Prove likewise that two triangles are equal, if three sides of one are equal respectively to the three sides of the other.

5. Prove that two triangles are equal, if they agree in one side and the two adjacent angles.

6. Are two triangles necessarily equal in size, if they agree in the three angles? Can any two such triangles be placed together so as to form a symmetrical quadrilateral? Why not?

Exercise 66.

1. Two points, A and B, are situated on opposite sides of a pond. If point C can be reached from A and B in straight lines, show how it is possible to obtain the distance between A and B by laying off $CE = AC$, $CD = BC$, and measuring ED (Fig. 44).

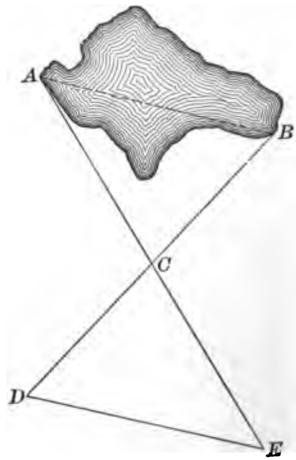


FIG. 44.

2. Draw any parallelogram and connect two of its opposite corners by a line. (This line is called a *diagonal*, as it goes "across corners.") Show how this diagonal divides the parallelogram into two equal parts.

3. What relation exists between opposite sides of a parallelogram? Make a statement about this and prove it.

4. What relation exists between opposite angles of a parallelogram? Make a statement about this and prove it.

5. Show that the diagonals of a parallelogram bisect each other.

Exercise 67.

1. Construct a triangle ABC, in which $AB = 2$ inches, $BC = 2\frac{1}{2}$ inches, and $AC = 3$ inches. Draw BD to bisect $\angle B$. Fold ABD over BD as an axis, locate the position of A, and call it A'. Unfold, and draw $A'D$.

(*a*) What is the relation of

(i) $\angle BA'D$ to $\angle C$? Why?

(ii) $\angle A$ to $\angle C$? (See Exercise 35, page 37.)

(*b*) Examine the relation existing between

(i) $\angle B$ and $\angle C$. (Bisect $\angle A$.)
(ii) $\angle A$ and $\angle B$. (Bisect $\angle C$.)

(*c*) Which is the largest and which is the smallest of the angles in the triangle?

2. What would be the relation between $\angle A$ and $\angle C$, if

(*a*) $AB = BC$?
(*b*) AB was greater than BC?

3. How can you tell at sight the relative magnitude of the angles in a triangle?

4. Which is the longest side in an obtuse triangle?

5. Which is the longest side in a right triangle?

6. Which is the shortest line from a point to a line?

Exercise 68.

Construct an isosceles triangle ABC (Fig. 45), when

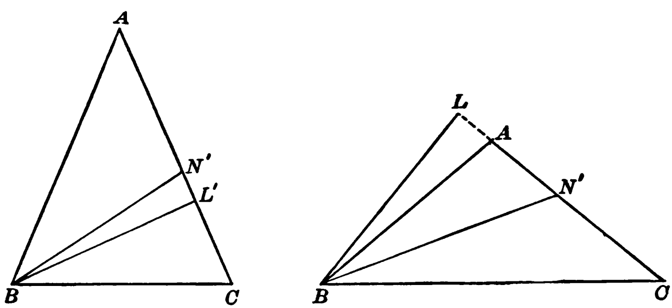


FIG. 45.

1. $BN' = 6$ centimetres and $\angle B = 45°$.

Make trial figure (see Exercise 36, page 39).

2. $BC = 3$ in. and $BL' = 2\frac{1}{2}$ in.

3. $BL' = 45$ mm. and $\angle A = \begin{cases} \text{(i) } 112\frac{1}{2}°. \\ \text{(ii) } 67\frac{1}{2}°. \end{cases}$

4. $BL' = a$ and $\angle B = m°$.

See note on page 30.

5. $BC = b$ and $\angle A = n°$.

Exercise 69.

1. In triangle ABC, BD is drawn to make $AD = AB$ (Fig. 46). What relation exists between DC and the sides AB and AC? What kind of a triangle is ABD?

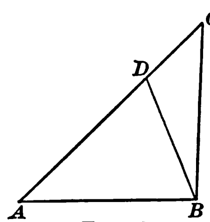


FIG. 46.

2. Construct a triangle equal to ABC (Fig. 46) and then find the line which is equal to $AC - CB$. Why can you not represent $(BC - AC)$ in this figure? $(AB - AC)$?

3. Draw a triangle ABC, in which you can represent:
(i) $(BC - AC)$; (ii) $(AB - AC)$.

4. Draw a triangle equal to BCD (Fig. 46). Without measuring DA, how would you construct the isosceles triangle DAB to complete triangle ABC?

See problem 1, Exercise 51, page 55.

5. Construct a triangle ABC, in which

(a) $(AC - AB) = 10$ mm., $BC = 45$ mm., and $BL' = 30$ mm.

For meaning of BL', see Exercise 37, page 41.

(b) $(AC - AB) = \frac{3}{4}$ in., $BL' = 2$ in., and $\angle A = 45°$.

(c) $(AC - AB) = d$, $BL' = l$, and $\angle C = p°$.

Exercise 70.

1. Draw a triangle, ABC. Extend BA to D, making $AD = AC$. Join C and D (Fig. 47).

(a) What kind of a triangle is DAC?

(b) What relation exists between BD and the sides AB and AC? (Write: $BD =$ ——.)

(c) What other similar construction can you make to represent the sum of AB and AC in a straight line?

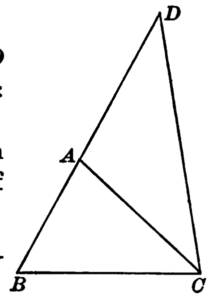


FIG. 47.

2. Draw any triangle, ABC, and represent in two ways:

(a) $AB + BC$; (b) $BC + AC$.

3. At what point would a perpendicular from A (Fig. 47) meet the line CD?

See problem 3 e, Exercise 48, page 54.

4. Draw a triangle equal to BCD (Fig. 47). Without measuring AB or AC, how would you find the point A, so that $AB + AC = BD$?

See problem 1, Exercise 51, page 55.

5. Construct a triangle ABC (Fig. 48), in which

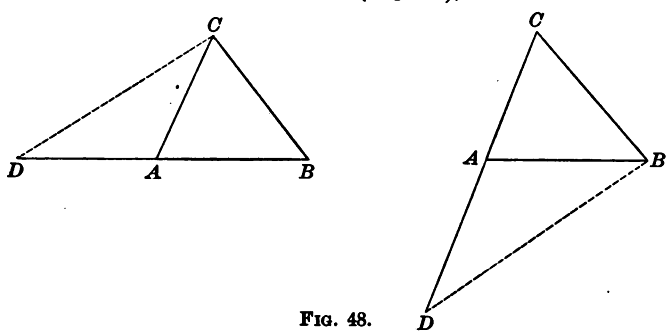


FIG. 48.

(a) $(AB + AC) = 3$ in., $BC = 2$ in., $\angle B = 60°$.

(b) $(AB + AC) = 100$ mm., $BC = 60$ mm., $\angle A = 45°$.

(c) $(AB + AC) = 4\frac{1}{2}$ in., $\angle A = 60°$, $\angle B = 45°$.

(d) $(AB + AC) = s$, $BC = a$, *$CL'' = h$.

(e) $(AB + AC) = s$, $CL'' = h$, $\angle A = m°$.

Exercise 71.

Construct a triangle, ABC, from lines and angles representing:

A.

1. $(BC + AC)$, AB, $\angle A$.
2. $(BC - AC)$, $\angle A$, $\angle C$.
3. $(BC - AB)$, AC, $\angle C$.
4. $(BC + AB)$, $\angle A$, $\angle B$.
5. $(BC + AB)$, AC, AL.
6. $(BC - AB)$, AC, AL.
7. $(BC - AC)$, AL, $\angle C$.
8. $(AC + AL)$, AB, $\angle C$.
9. $(AC + AM)$, BC, $\angle C$.
10. $(AB + AN)$, $\angle A$, $\angle B$.

B.

11. $(BC + AC)$, AB, $\angle B$.
12. $(BC - AC)$, $\angle B$, $\angle C$.
13. $(BC - AB)$, AC, $\angle B$.
14. $(BC + AB)$, $\angle A$, $\angle C$.
15. $(BC + AB)$, AL, $\angle B$.
16. $(BC - AB)$, AL, $\angle B$.
17. $(BC - AC)$, AB, AL.
18. $(AC + CL'')$, BC, $\angle A$.
19. $(BC + BM')$, AC, $\angle C$.
20. $(BC + BN')$, $\angle A$, $\angle B$.

Select any lines and angles to represent the parts stated in the problem. Then proceed as outlined in Exercise 36, page 39. For meaning of names, CL'', etc., see Exercise 37, page 41.

---

* See note in Ex. 37, page 41.

Exercise 72.

1. Supposing your teacher has drawn a parallelogram on the board, and you are required to construct one equal to it; how many points will you have to locate? Will you have to measure all the sides and angles to do this? What is the least number of parts that you will have to measure? Tell how you can build up the parallelogram from the parts you have measured.

Let the pupils suggest different ways.

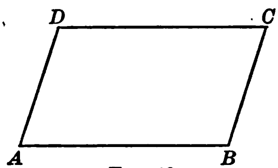


FIG. 49.

2. Construct a parallelogram, *ABCD* (Fig. 49), when

(*a*) $AB = 2$ in., $AD = 1$ in., and $\angle BAD = 45°$.

(*b*) $AB = 50$ mm., $AC = 70$ mm., and $\angle BAC = 20°$.

(*c*) $AB = 2\frac{1}{2}$ in., $AD = AB$, and $\angle BCD = 60°$.

(*d*) $AD = 30$ mm., $BD = 50$ mm., and $\angle ADB = 25°$.

(*e*) $AB = a$, $AD = b$, and $AC = e$.

(*f*) $CD = a$, $BC = b$, and $AC = e$.

Always sketch the kind of figure you are to make and note the relative positions of the parts corresponding to those given you with respect to the points you want to locate. This will aid you in determining how to go about your work. (See Exercise 36, page 39.)

3. Can you reproduce a certain parallelogram, if you know

(*a*) All its sides? Why not?
(*b*) All its angles? Why not?

4. How do you construct a rectangle, when you know the two unequal sides? What relation of size do you notice between its diagonals?

5. How do you construct a rhombus, when you know one of the sides and one of the angles? (Make a trial figure.)

6. What kind of an angle is formed by the diagonals of a rhombus? Why?

See problem 1 *c*, Exercise 57, page 59.

Exercise 73.

Construct a parallelogram, $ABCD$ (Fig. 50), having given:

(*a*) AD, AB, and BD.
(*b*) AD, BD, and $\angle ADB$.
(*c*) AB, BL', and BD.
(*d*) AD, AC, and BD.

See problem 5, Exercise 66, page 68.

(*e*) AC, $\angle CAB$, and $\angle AOB$.
(*f*) AD, BD, and AL.

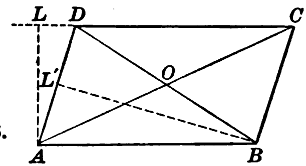


FIG. 50.

What kind of a parallelogram will you get, if you select $AD = AL$? What happens if AD is selected less than AL?

Exercise 74.

1. Construct a trapezoid, $ABCD$ (Fig. 51), from:

(*a*) AD, AB, CD, and $\angle A$.
(*b*) BC, AD, BD, and $\angle CBD$.
(*c*) AB, AD, $\angle D$, and $\angle CBD$.

HINT. What is the relation between $\angle D$ and $\angle A$?

(*d*) AB, CD, AD, and DL.

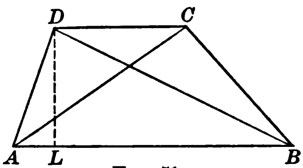


FIG. 51.

2. Construct an isosceles trapezoid, $ABCD$ (Fig. 52), in which $AD = BC$, from measurements for:

(*a*) AB, AD, and AC.
(*b*) AB, BC, and DL.
(*c*) AB, $\angle A$, and $\angle CAB$.
(*d*) AC, $\angle CAB$, and $\angle CBD$.

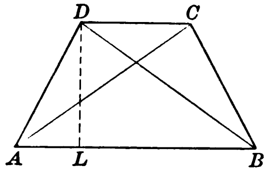


FIG. 52.

Exercise 75.

1. Draw an angle with the vertex O, and on one of its arms measure $OA = AB = BC$ (Fig. 53). Join C to any point, D, of the other arm, and draw AE and BF parallel to CD, and AGH and BI parallel to OD.

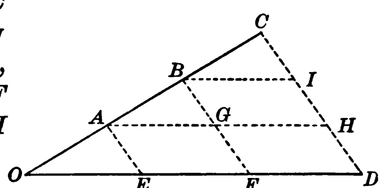


FIG. 53.

(*a*) What relation do the triangles OEA, AGB, and BIC hold to one another? Why?

See problem 5, Exercise 65, page 67.

(*b*) What kind of a figure is $AEFG$? $BGHI$? $FDHG$? Why?

(*c*) What is the relation of triangle OEA to figure $AEFG$? How can you show it?

See problem 2, Exercise 66, page 68.

(*d*) Why is $OE = EF = FD$?

(*e*) What way of dividing a given line into three equal parts can you derive from this?

2. Likewise, divide a line into five equal parts.

3. Divide a three-inch line into five equal parts. What is the length of each one of the parts?

4. Draw a line five-sevenths of an inch long.

5. Given the triangle ABC, and D, the middle point of AC. If a line is drawn through D parallel to AB and cutting BC, how will that line divide BC? Why? Make a statement about this.

6. Draw any triangle and join the middle points of its sides, dividing the triangle into four parts. What is the relation of the parts to each other? To the triangle?

7. In Fig. 54, what is the relation, or *ratio*, of AA' to BB'? AA' to PQ? BB' to PQ? CC' to PQ?

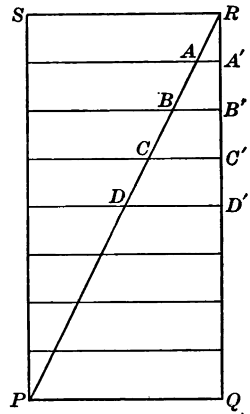


FIG. 54.

8. Figure 55 is a device known as the *decimal scale.* It shows CA, the unit of measure, in tenths and hundredths, so that the distance between AB and XY measured on the top lines is 4 tenths of the unit, measured on the next, or second, line 41 hundredths, on the third 42 hundredths, etc. Show how and why?

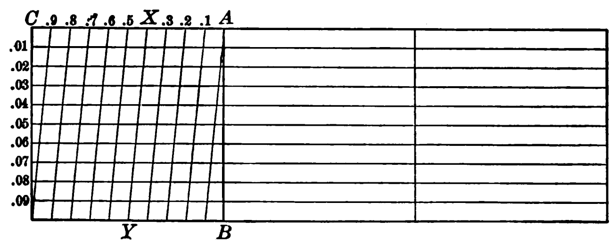


FIG. 55.

9. Construct a line equal to 1.55 inches; 2.78 inches; 1.06 inches.

10. Draw lines at random and measure their lengths correct to one hundredth of an inch by the decimal scale.

CHAPTER VIII.

DRAWING TO SCALE.

Exercise 76.

1. Is it possible to draw an object which is 6 feet by 2 feet, on an ordinary piece of drawing-paper ($8\frac{1}{2}$ inches by 11 inches)? How is it done? What kind of relation would exist between the object and the picture?

2. What is a plan of a house? How can the carpenter or the builder obtain correct dimensions from a plan?

3. Draw a line 1 inch long, and let it represent a distance of 8 feet. What distances would be represented by $\frac{1}{2}$ inch? 3 inches? $3\frac{1}{16}$ inches?

Lines thus drawn to represent lengths other than their own are said to be *drawn to a scale;* in this case, the scale is 1″ to 8′. (Read "1 inch to 1 foot.")

4. Draw to the same scale: 4 feet; 16 feet; 5 feet.

5. Draw a line 4 inches long and let it represent a distance of 28 feet. To what scale is this line drawn?

6. Represent $3\frac{1}{2}$ feet; 21 inches; 35 feet.

7. Draw by a scale of $\frac{1}{4}$ of an inch to a foot a line 2 feet long; a line 7 feet long; a line 6 feet 9 inches long.

8. Draw by a scale of $\frac{1}{4}$ of an inch to a foot the plan of a room 16 feet long and 14 feet wide.

Exercise 77.

1. Represent — in convenient size — a triangular lot whose sides are respectively, 100 yards, 50 yards, and 75 yards.

This process of repeating on paper, to a smaller scale, the measurements which have been made on the ground, is called by surveyors *platting;* and the resulting figure is called a *plat.*

2. A piece of land extends northeast 160 yards, then northwest 40 yards, then southwest 160 yards, and lastly southeast 40 yards. Draw a plat of the land.

3. A stone dam, vertical on one side and slanting on the other, is 60 feet long, 12 feet wide at the bottom, and 4 feet wide at the top. Draw to scale a cross-section of the dam and from the drawing determine the length of its slant height. (Measure with a decimal scale.)

4. A rectangular cricket field is 70 yards long and 50 yards wide. Make a plat of the field.

5. A flagpole, 84 feet in height, was partly broken in a storm, the top falling over and resting on a stone which is 32 feet from the foot of the pole. Draw a diagram to a scale of $\frac{1}{8}$ inch to 4 feet, and from it determine the length of the part broken off.

See Exercise 70, page 71.

6. Draw a plat which will represent a rectangular piece of land, 300 feet long and 200 feet wide, divided into lots 25 feet by 100 feet.

7. Two sides of a triangular garden-plot are 24 and 32 feet, respectively. The angle included by them is 60°. Draw to a convenient scale and determine the length of the third side.

8. A man at R (Fig. 56) desired to know the distance between two inaccessible points, P and Q, which are in a line with his position at R. So he located another point, A, a distance of 520 yards from R, and then measured* the angles $ARQ = 25°$, $RAQ = 30°$, and $QAP = 55°$. He then made a plat and from it determined the distance PQ. Scale: $\frac{1}{8}$ in. = 40 yd.

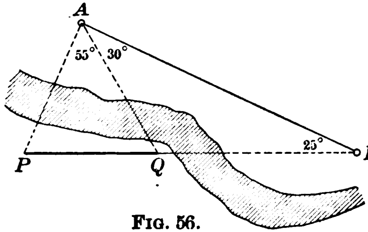


FIG. 56.

(a) Using Fig. 56 to represent the plat, show how it can be made, and find the length of PQ.

See Exercise 36, page 39.

(b) Find the length of PQ if $\angle ARQ = 45°$, $\angle RAQ = 52\frac{1}{2}°$, and $\angle QAP = 60°$, the distance AR remaining the same.

9. A tree, T, and a straight road AB, 1 mile long, are situated on opposite sides of a pond (Fig. 57). At A, a line from A to the tree is observed to make an angle of 45° with the road, and at B, a line from B to the tree is found to make an angle of 30° with the road.

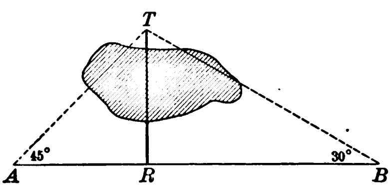


FIG. 57.

(a) From the plat (Fig. 57), find TR, the perpendicular distance of the tree from the road.

(b) Find the distance TR, if $\angle A = 37\frac{1}{2}°$ and $\angle B = 52\frac{1}{2}$.

---

* Surveyors measure such angles with an instrument, called *transit*. For rough measurements, however, a large protractor and a flat ruler with a thin edge will serve the purpose, the ruler being used as the sight.

10. Two lighthouses at the mouth of a harbor are each two miles from the wharf. A person on the wharf finds the angle between the lines to the lighthouses to be $22\frac{1}{2}°$. Make a plat and find the distance between the two lighthouses.

11. P and Q are on opposite sides of a stream. Show by construction what measurements may be made to determine the distance between the points without crossing the stream (Fig. 58).

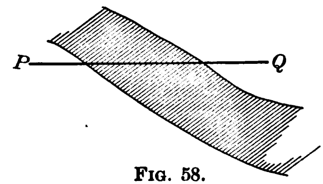


FIG. 58.

12. To find the length of an inaccessible line, PQ (Fig. 59), a surveyor places his transit first at A then at B, measuring the angles BAQ, QAP, QBP, and PBA. Draw a plat and determine PQ if

(a) $AB = 100$ yards;
$\angle BAQ = 65°$;
$\angle QAP = 55°$;
$\angle QBP = 50°$; and
$\angle PBA = 45°$.

(b) $AB = 100$ yards;
$\angle BAQ = 90°$;
$\angle QAP = 50°$;
$\angle QBP = 50°$; and
$\angle PBA = 30°$.

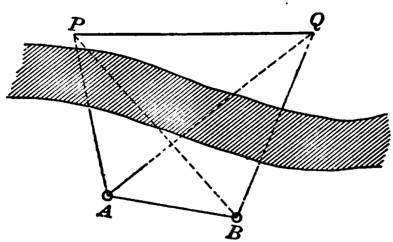


FIG. 59.

HINT. Construct triangles ABP and ABQ from given measurements drawn to scale.

13. A rope, 120 feet long, stretched from the top of a flagpole makes an angle of $67\frac{1}{2}°$ with the ground. Draw to scale and determine the height of the pole from your drawing.

14. To determine the height of a tower, a boy places his angle measurer first at A and then at B, two points a convenient distance apart and in line with the tower, and measures the angles QAR and QBR (Fig. 60). (These

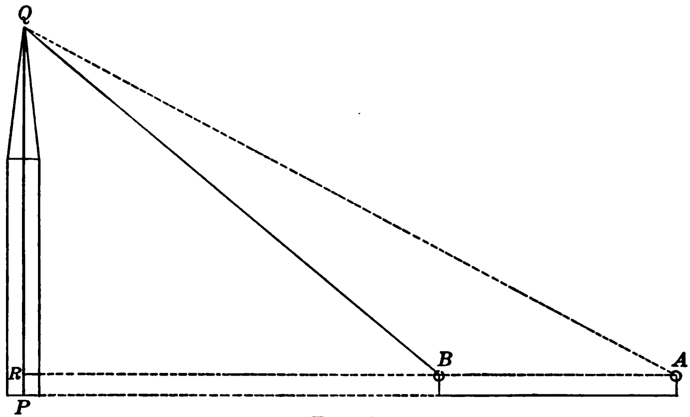


FIG. 60.

angles are called *angles of elevation.*) The angle measurer stands 4 feet above the ground. Show by a plat, how the boy may obtain the height, if $AB = 50$ feet; $\angle QAR = 30°$; and $QBR = 60°$.

15. Determine the height of some building in your neighborhood, making similar measurements.

16. At a certain point the angle of elevation of a tower was observed to be 60°, and at a point 300 feet farther away in the same straight line, it was 15°. Find the height of the tower.

17. The shadow of a building, 100 feet high, is found to be 125 feet long. Find the angle of elevation of the sun at that time.

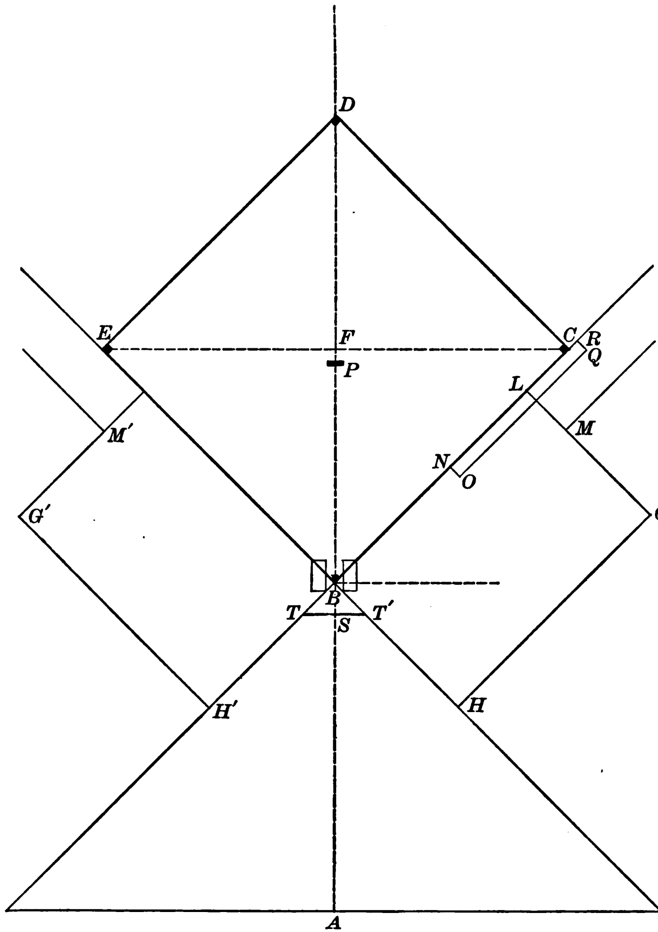


FIG. 61.—Diagram of a Base-ball Field.

Exercise 78.

1. The plan of a baseball field (Fig. 61) is drawn $\frac{3}{32}$ inch to 5 feet. Find the lengths of all the lines in the drawing, and then give from it the directions for laying out a ball ground. Is the "diamond" a square or a rhombus?

2. To lay out a tennis-court for the single game, you mark off a rectangle 78 feet by 27 feet. On each one of the long sides stake out the points X, Y, and Z, distant respectively 18, 39, and 60 feet from either one of the short sides. Connect the middle points of XX and ZZ, and stretch the net across at YY. The double court is made by extending the short sides $4\frac{1}{2}$ feet each way and connecting their extremities, thereby forming a larger rectangle. Draw:

(*a*) A plan for the single court.
(*b*) A plan for the double court.

3. The boundary of a lot is as follows: Beginning at a point A, it runs 8 rods east to a point B, thence 16 rods north to C, thence 24 rods west to D, thence 32 rods south to E, and from there to A. Draw a plan of the lot by a scale of $\frac{1}{8}$ of an inch to a rod. Find the number of feet of fence required to enclose the lot.

4. The owner of a piece of land 790 feet long and 540 feet wide cuts two streets, each 40 feet wide, through the middle of the land, one running north and south, the other east and west. The rest he divides into building lots 25 feet by 100 feet. Draw a plat of the land.

5. The owner of a piece of ground, 235 feet wide and 320 feet long, divides it into lots 25 feet by 100 feet, reserving 10 feet all around for a walk, and a strip 15 feet wide and running lengthwise through the middle of the ground, for an alley. How many lots are there? Show by making a plat.

Exercise 79.

1. What is a map? How can you ascertain from it the distances between places? If the scale to which a map is drawn is 1 inch to 200 miles, and the distance between two cities on the map is $4\frac{1}{2}$ inches, what is their actual distance from each other?

2. The distance between New York and San Francisco is about 3200 miles. How far apart would these places be on a map which is drawn to a scale of 1 inch to 200 miles? Judging from this, what do you estimate the scale of the map of North America in your Geography to be?

3. A certain country is 720 miles long and 400 miles wide. Find what size your paper must be so you can draw a map of this country on it to the scale: $\frac{1}{16}$ inch $=$ 1 mile.

4. If $\frac{3}{8}$ inch on a map corresponds to an actual distance of 9 miles, what distance on the map represents 21 miles?

5. Scales are often expressed fractionally to indicate what part of the real line, measured on the ground, the representative line actually is. In the United States Engineer Service, general plans of buildings are drawn to the scale: 1 inch to 10 feet. This is expressed: Scale $\frac{1}{120}$, which means 1 inch to 120 inches, or 1 inch to 10 feet. Express fractionally the scales:

(*a*) 1 inch to 1 yard. (*c*) $\frac{1}{8}$ inch to 100 feet.
(*b*) 1 inch to 1 mile. (*d*) 1 mm. to 1 km.

6. What scale may be expressed by:

(*a*) $\frac{1}{15840}$? (*b*) $\frac{1}{600}$? (*c*) $\frac{1}{5280}$? (*d*) $\frac{1}{10560}$? (*e*) .001?

7. On a map, drawn to scale $\frac{1}{2640}$, how long a line represents a distance of 1 mile? A distance of 1000 yards?

Exercise 80.

1. This diagram of a basket-ball ground (Fig. 62) is drawn to a scale such that MN represents a distance of 15 feet. Determine the scale, and find the lengths represented by the other lines. What directions would you give for laying out the ground?

2. Ascertain the correct dimensions, and then make a plan of a football ground.

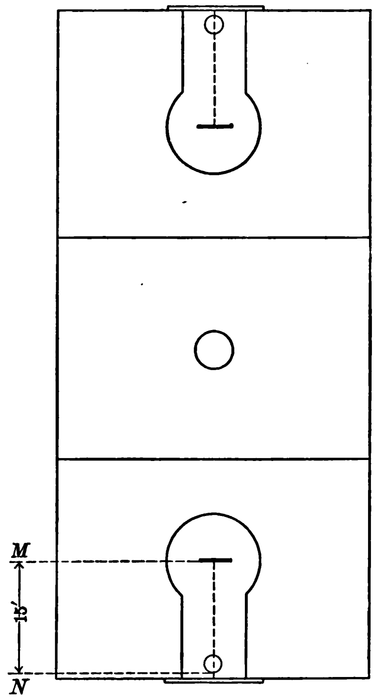


FIG. 62.—Diagram of a Basket-ball Field.

Exercise 81.

If one boy can throw a ball 50 feet, while another can throw it 75 feet, then the distances, 50 feet and 75 feet, indicate the relative strengths of the boys or measure the force with which each can throw the ball. These distances may be represented by lines and drawn to a scale, and the lines may be placed so as to show also the direction of each throw.

1. Suppose one boy throws a ball from P to Q, a distance of 50 feet to the north from P, and another picks it up at Q and throws it east to R, a distance of 75 feet, what line will represent the direction and intensity of the single throw which will bring the ball direct from P to R?

Draw a diagram to a convenient scale, and determine the length of PR.

2. Two forces are pulling a weight, W, at an angle of 45° to each other, with an intensity of 9 pounds and 17 pounds, respectively (Fig. 63).

(*a*) Suppose the forces to act one after the other, in how many ways may the weight reach its destination?

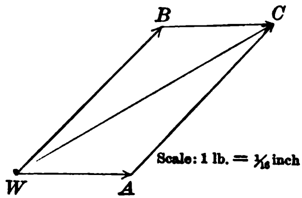


FIG. 63.

(*b*) Suppose the forces to act together, what line will represent the direction and intensity of the resulting force? What kind of a figure is formed by the lines representing the forces? How large is the force represented by WC?

3. What is the intensity of the resulting force, if the forces are each 16 pounds, pulling at an angle of 60°? Draw to a scale like Fig. 63 and measure WC.

4. A man is rowing at the rate of 6 miles an hour across a stream flowing at the rate of 3 miles an hour. The river is 2 miles wide. How long will it take the man to cross, and how far down will he land?

5. A man swims at right angles to a stream at the rate of 3.2 miles an hour. The rate of the current is 8 miles an hour. Construct the line which represents the rate of the man's motion through the water and compute its value. If the stream is 528 feet wide, how far down will the swimmer be carried? (Draw to the scale: $\frac{1}{2}$ inch = .8 mile.)

6. A ship is making 15 miles an hour on a southeast course. How fast is she moving east? How fast is she moving south? Draw to a scale and compute from diagram.

Exercise 82.

Drawing to scale may be used to advantage in other problems. Note, for instance, how the following graphic representation (Fig. 64) of the numbers in the table of statistics on page 88 adds to an understanding of the facts.

GROWTH OF POPULATION IN THE UNITED STATES BY DECADES.

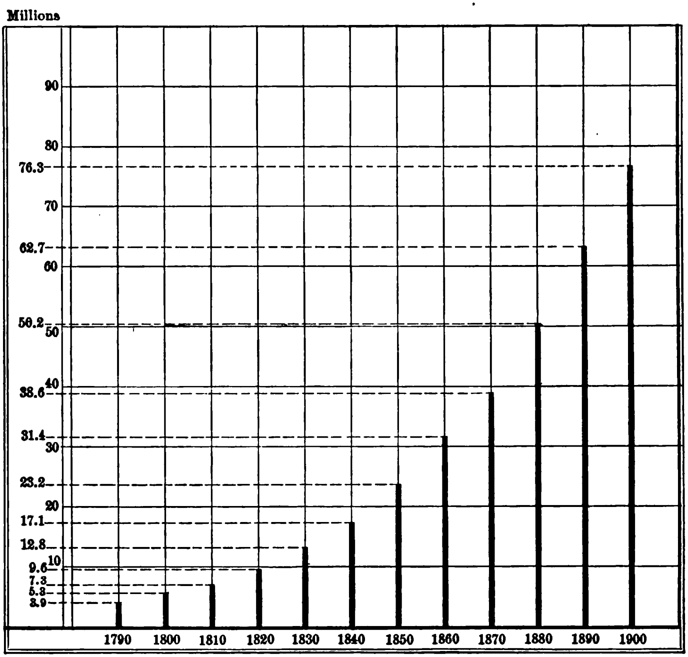
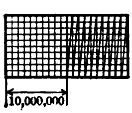


FIG. 64.



Scale.

Every half inch of the heavy vertical lines represents 10,000,000 people.

The first census of the United States was taken in 1790, and there has been one taken every tenth year since that time. The following table shows the number of inhabitants, "excluding Indians not taxed," at each decennial period.

YEAR.	POPULATION.	YEAR.	POPULATION.
1790	3,929,214	1850	23,191,876
1800	5,308,433	1860	31,443,321
1810	7,229,881	1870	38,558,371
1820	9,663,822	1880	50,155,783
1830	12,806,020	1890	62,669,756
1840	17,069,453	1900	76,295,220

A diagram, or *graph*, like Fig. 64 (page 87) may be used whenever we wish to compare different measurements or values, as in the following examples.

1. A pound of wood, when burned, will heat 7000 pounds of water one degree. On the same basis, wax will heat 19,000 pounds of water, phosphorus 10,350, coal gas 22,500, hydrogen 62,000, fats 17,460, coal oil 18,000, bituminous coal 14,700, anthracite 10,800, pure carbon 14,500. Arrange in order and represent these measurements by a diagram.

2. The specific gravity of a solid is the ratio of its weight to the weight of an equal volume of water. Thus, 7.86, the specific gravity of iron, expresses the fact that, bulk for bulk, iron is 7.86 times heavier than water. Represent graphically the following specific gravities:

Water	1.00	Copper	8.82
Aluminum . . .	2.56	Silver	10.53
Zinc	7.15	Lead	11.37
Tin	7.29	Mercury	13.59
Iron	7.86	Gold	19.32

3. Represent graphically the "average precipitation" according to the statistics in the annexed table:

TOTAL PRECIPITATION (IN INCHES) IN CHICAGO.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
For 1901	1.15	2.05	3.38	.33	2.18	2.42	4.25	2.00	2.92	1.29	.85	1.70
For 1902	.66	1.53	4.16	2.26	5.08	6.45	5.78	1.44	4.83	1.45	2.03	1.90
Average for 32 years	2.07	2.30	2.64	2.70	3.56	3.79	3.61	2.83	2.91	2.63	2.75	2.71

Exercise 83.

Figure 65 is drawn to show how the precipitations from month to month in the years 1901 and 1902 compare with the average

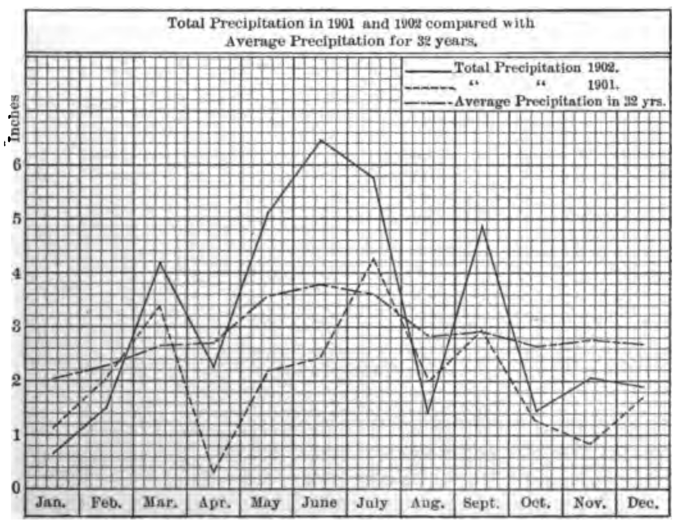


FIG. 65.

precipitation computed for a period of 32 years. Each of the three broken lines, or *curves*, is made by connecting the top-points of a set of vertical lines, like those in Fig. 64, which belong to the same set of statistics, but which have not been drawn out for the reason that they would interfere with the corresponding lines of the other two sets.

The work of constructing these curves is called *plotting.*

1. Interpret Fig. 66. Make a thermometer record for one day of the current week.

THERMOMETER–RECORD FOR ONE DAY.

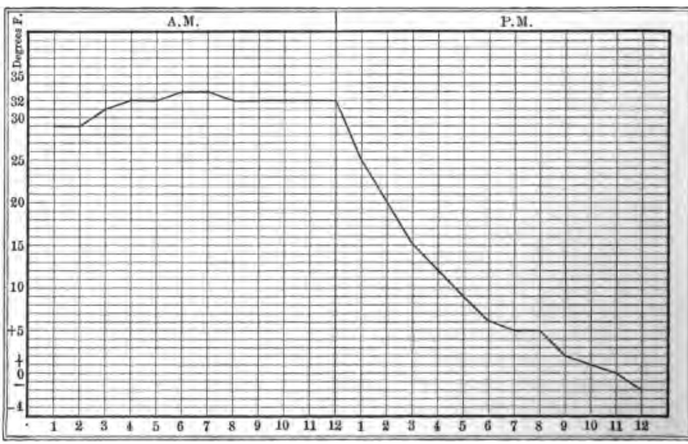


FIG. 66.

2. Keep a record of the temperature for the hours from 6 A.M. to 6 P.M. of every day of this week. Then plot the curves for the two days differing most in temperature and the curve for the average hourly temperature of the week.

3. The maximum, minimum, and mean temperatures for Chicago during the months of the year 1902 are as follows:

	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Maximum	50	48	65	83	86	91	90	86	81	76	70	50
Minimum	−8	−7	3	23	36	48	54	55	42	35	26	−1
Mean	25	21	39	46	59	64	73	68	61	55	47	26

Plot the three curves corresponding to these statistics and note where the range of temperature is greatest.

4. Plot the temperature curves, from statistics in appended table, for Boston, St. Louis, and Los Angeles. Note the differences due to latitude and other climatic influences.

Station	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Boston	26.9	28.3	34.1	44.4	56.8	66.4	71.9	69.9	63.1	52.1	41.1	31.5
Chicago	24.0	27.8	34.4	46.2	56.2	66.4	72.0	70.8	64.2	52.6	39.0	30.4
Key West	70.6	72.2	73.0	76.4	79.6	83.0	84.4	84.1	82.8	78.9	74.7	70.5
Los Angeles	53.0	54.7	57.3	60.0	63.5	67.4	71.6	72.4	70.2	64.4	59.6	55.7
Montgomery	48.6	53.3	57.6	65.8	73.4	80.3	82.6	80.8	74.3	65.7	55.6	49.9
Moorhead	−1.0	3.8	19.4	41.4	53.1	64.4	67.5	65.4	56.4	43.4	25.2	11.8
New York	30.8	32.0	36.6	48.3	59.8	69.2	74.2	72.8	66.5	55.4	44.1	34.6
St.Louis	31.4	36.1	43.2	56.3	65.7	74.8	79.3	76.9	69.6	57.9	44.3	37.8
San Antonio	51.0	56.6	62.2	69.6	75.0	81.1	83.7	82.9	77.6	69.9	58.8	54.9
Yuma	53.4	58.3	64.6	70.2	77.4	84.8	92.0	91.4	85.0	72.9	61.8	56.4

Extract from Weather Bureau Report of Normal Temperatures.

5. Plot the normal temperature curves for Yuma, Ariz., and Montgomery, Ala., which are about on the same parallel of latitude.

6. Plot the normal temperature curves for Moorhead, Key West, and San Antonio.

7. Plot the curves for New York and Chicago.

8. The premiums for fire-insurance and the losses for the years 1891 to 1901 in Chicago were:

Years.	Net Premiums.	Losses.
1891	$4,251,975	$3,292,046
1892	4,578,897	1,640,257
1893	4,428,392	3,679,697
1894	5,358,452	4,717,949
1895	5,416,578	3,877,296
1896	5,302,117	2,313,634
1897	6,086,067	3,708,076
1898	5,936,578	4,071,711
1899	6,835,181	5,740,058
1900	6,977,096	3,080,054
1901	7,537,507	4,614,870

Represent by two curves.

9. Procure an almanac giving the times of sunrise and sunset for different latitudes, — the latitudes of Boston and Charleston, for instance, — and compute the corresponding lengths of days at intervals of two weeks throughout a whole year, beginning with the first Monday of the year. Then represent your results graphically. Note the differences in the two curves.

What kind of a curve would you obtain for a polar latitude? A latitude near the equator?

Exercise 84.

1. Draw a triangle having two of its sides equal to 2 and 3 inches respectively and the angle included by them equal to 60°
   (*a*) Actual size.
   (*b*) To the scale $\frac{1}{2}$ inch to 1 inch.

These triangles, though different in size, are *equal in form.* Their corresponding angles are equal.

Triangles so related are said to be *similar.*

2. Examine Figs. 53 and 54 and pick out two pairs of similar triangles. What is the relation of their sides?

3. Draw any triangle, and then make another one similar to it, whose sides are $\frac{1}{5}$ as long. (See Exercise 75, page 75.)

4. *A* wants to determine his distance from another man, *B*, knowing *B* to be 70 inches tall. He holds a foot-rule at arm's length, a distance of 2 feet from the eye, and finds that the man's height covers 1 inch (Fig. 67). How far is he from *B*?

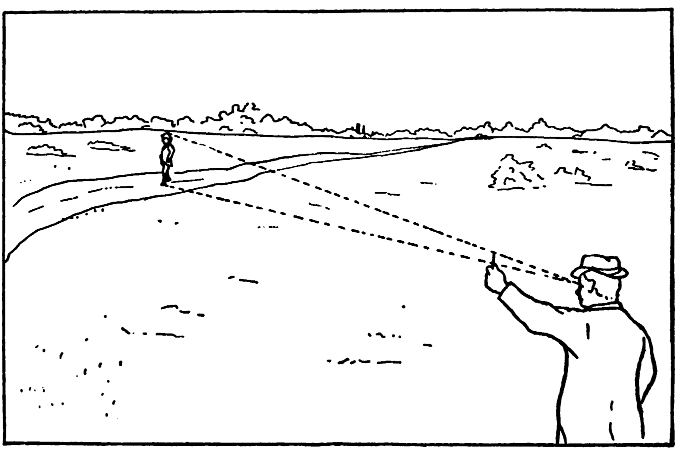


Fig. 67.

5. Woodchoppers, desiring to know the height of a tree before cutting it, sometimes make an isosceles right triangle of wood or paper, and step off the distance on level ground

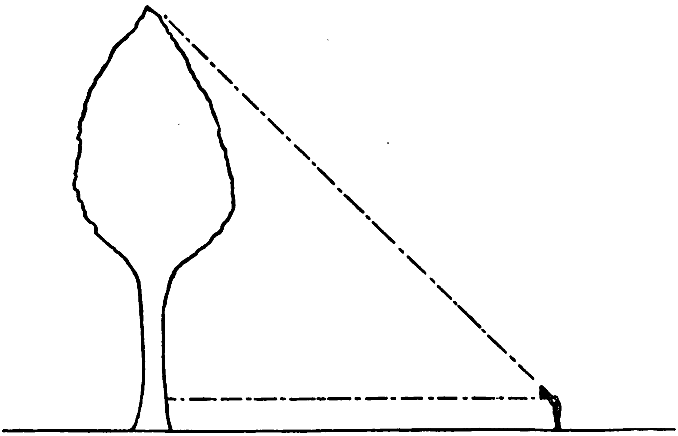


Fig. 68.

from the foot of the tree to a point at which they find they can just see its top when looking along the hypotenuse of the triangle, held with one side parallel to the ground. Show why they thus obtain a pretty correct estimate (Fig. 68).

6. Two poles of unequal lengths are so placed as to throw their shadows on a level piece of ground, the smaller, which is 6 feet long, throwing a shadow of $7\frac{1}{2}$ feet, and the taller, one of 60 feet.

Draw to a convenient scale. How can you obtain the height of the taller?

7. An upright post (11 feet long) casts a shadow of $9\frac{1}{8}$ feet; what will be the height of a church tower which casts a shadow 224 feet long?

8. The Washington monument casts a shadow 111 feet when a post 6 feet high casts a shadow 14.4 inches. Find the height of the monument.

9. A boy, whose eye is 4 feet from the ground, can just see the top of a flagpole when looking over a fence which

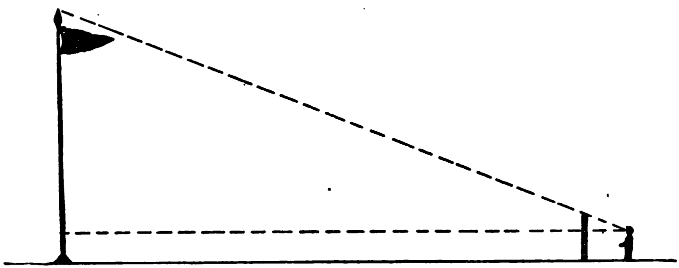


FIG. 69.

is 6 feet high and 3 feet away from him (Fig. 69). The distance of the flagpole from the fence is 168 feet. What is its height?

Exercise 85.

1. Draw any irregular pentagon, *ABCDE*, and all the diagonals you can from the vertex *A*. From *F*, any point in *AB*, make *FG* parallel to *BC*. Then draw *GH* parallel to *CD*, and *HI* parallel to *DE* (Fig. 70). What relation exists between *AFGHI* and *ABCDE*? If *F* were the middle point of *AB*, what would be the relation, or ratio, of the corresponding sides? (See Exercise

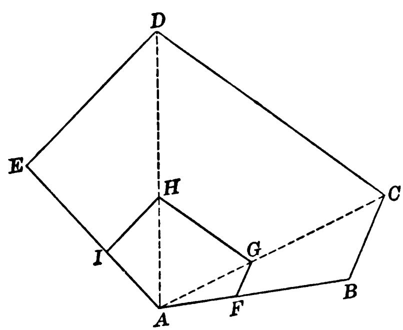


FIG. 70.

75, page 75.) How would you have to divide AB to obtain a polygon similar to $ABCDE$ and with sides $\frac{1}{5}$ as long?

2. Draw any rectangle and construct another one similar to it with sides:

(*a*) Half as long.
(*b*) Twice as long.
(*c*) Three-fourths as long.

3. Do likewise with:

(*a*) A rhombus having one of its angles equal to 45°.
(*b*) A hexagon.

Exercise 86.

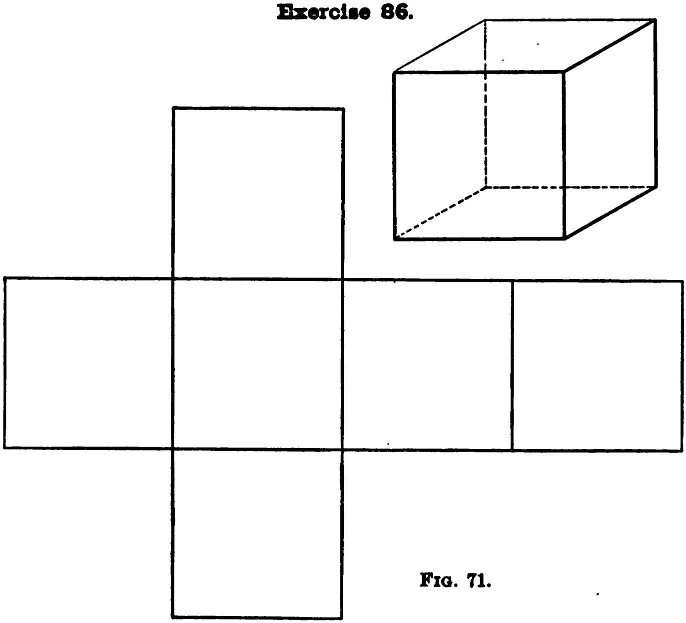


FIG. 71.

1. Place a cube on a sheet of paper and, turning it over and over on its edges from face to face, trace each face as it comes in contact with the paper (Fig. 71).

This is called *developing* the surface of the cube. The resulting sketch is a sort of flat pattern from which we can obtain what we may call a *model* of the cube.

2. Can this diagram serve as a pattern to make a 2-inch cube? A 5-inch cube? If so, what length would each line of the drawing have to represent?

3. Can you develop the surfaces of other solids the same way? What about those having curved surfaces? The cylinder? The cone? The sphere?

4. Develop the surface of a given cylinder.

5. Develop the surface of a given cone.

Exercise 87.

1. Draw the developed surface of solid *1* (Chart VII), when the base $ABCD$ is a square with a side equal to 1 inch and the altitude AE is 3 inches.

2. Draw the developed surface of solid *3* (Chart VII), when the altitude AE is 3 inches and the base ABD is a triangle with sides of 1, $1\frac{1}{2}$, and 2 inches, respectively.

3. Draw to scale the developed surface of solid *1* (Chart VII), when $AB = 8$ inches, $AD = 4$ inches, $AE = 6$ inches,* and all angles are right angles.

4. Draw to scale the developed surface of solid *1* (Chart VII), when $AB = 11$ inches, $AD = 7$ inches, $AE = 3$ inches, and all angles are right angles.

5. Draw to scale the developed surface of solid *2* (Chart VII), when $AB = 9$ inches, $AD = 3$ inches, $AE = 6$ inches, $\angle BAD = 90°$, $\angle BAE = 60°$, and $\angle EAD = 90°$.

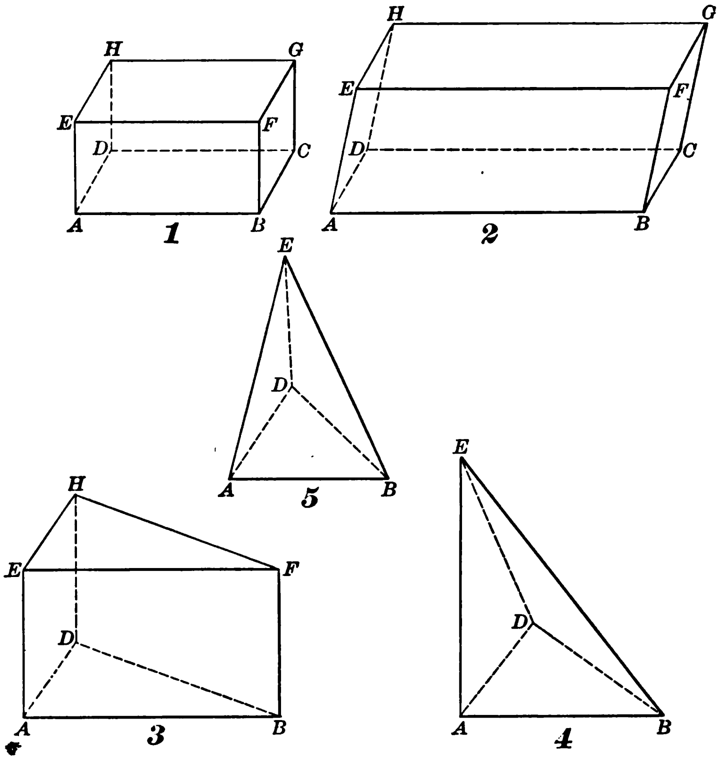


CHART VII.

6. Draw to scale the developed surface of solid *2* (Chart VII), when $AB=30$ inches, $AD=10$ inches, $AE=20$ inches, $\angle BAD = 90°$, $\angle BAE = 60°$, and $\angle EAD = 45°$.

7. Draw to scale the developed surface of solid *2* (Chart VII), when $AB = 5$ inches, $AD = 5$ inches, $AE = 5$ inches, $\angle BAD = 60°$, $\angle BAE = 60°$, and $\angle EAD = 60°$.

8. Draw to scale the developed surface of solid *3* (Chart VII), when $AB=9$ inches, $AD=12$ inches, $AE=20$ inches, $\angle BAD = 90°$, $\angle BAE = 90°$, and $\angle EAD = 90°$.

9. Draw to scale the developed surface of solid *4* (Chart VII), when $AB = 12\frac{1}{2}$ inches, $AD = 50$ inches, $AE = 62\frac{1}{2}$ inches, $\angle BAD = 90°$, $\angle BAE = 90°$, and $\angle EAD = 90°$.

10. Draw to scale the developed surface of solid *5* (Chart VII), when $AB = 9$ inches, $AD = 9$ inches, $AE = 9$ inches, $\angle BAD = 60°$, $\angle BAE = 60°$, and $\angle EAD = 60°$.

11. The dimensions of an ordinary building brick are 8, 4, and 2 inches. Make a model of one out of a piece of cardboard (5 inches by 11 inches).

12. (*a*) Design a box which can be made out of one piece of paper (12 inches by 12 inches).

(*b*) Design one with a lid attached to it.

Exercise 88.

1. Supposing you have a solid cylinder, 2 inches high and 2 inches wide, and you develop its surface on a flat piece of paper (Fig. 72); how are the lines of the cylinder represented in the drawing?

Will this help you — as a pattern — to make a cylinder of different dimensions? To make this drawing without using the solid, what must you know about it? Can you make it without knowing the length of the circumference?

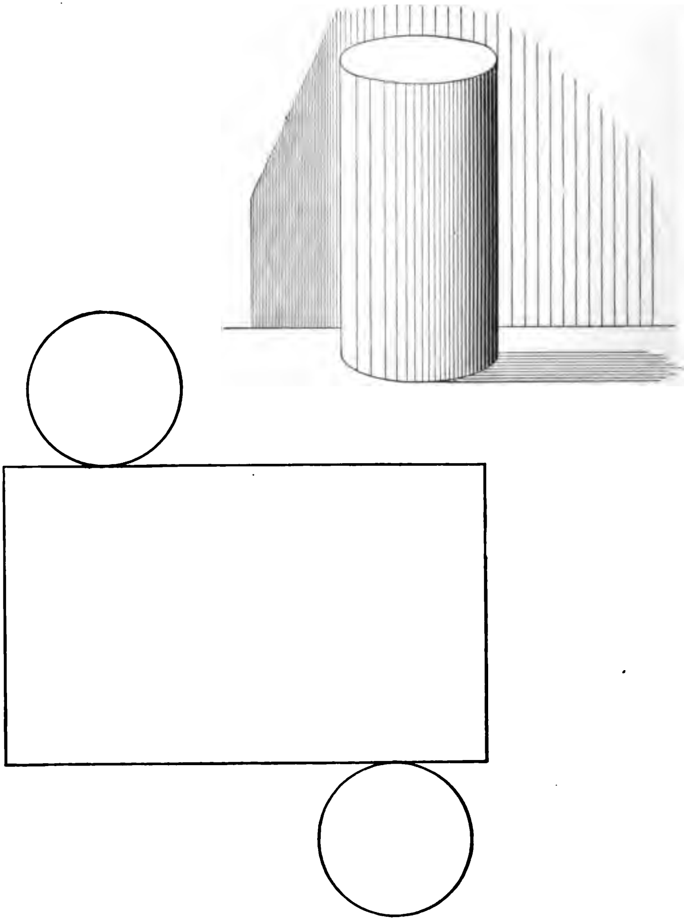


FIG. 72.

2. Is there a relation between the diameter of a circle and its circumference?

(*a*) Draw on cardboard a circle with a radius of 7 inches. Cut it out carefully and measure its circumference with a tape-line. Do likewise with circles of radii $5\frac{1}{2}$ inches and $1\frac{3}{4}$ inches.

Tabulate your results as follows:

	Radius	Circumference
(1)	7 in.	
(2)	5-1/4 in.	
(3)	1-3/4 in.	

(*b*) What is the relation, or ratio, of the circumference to the diameter in (1)? In (2)? In (3)?

(*c*) Judging from this, what should be the length of the circumference of a circle with a diameter of 1 inch? 2 inches? 5 inches? Verify by measuring.

Exercise 89.

Draw to a convenient scale a diagram for the construction of models of cylinders with:

	Width	Height
(*a*)	7 in.	8 in.
(*b*)	5 in.	5 in.
(*c*)	6-1/8 in.	1 in.
(*d*)	75 mm.	60 mm.

Exercise 90.

1. Develop the curved surface of a cone (Fig. 73). Prove that it is a part of a circle. (See Exercise 1, page 1.)

A part of a circle bounded by two radii and an arc, is called a *sector.*

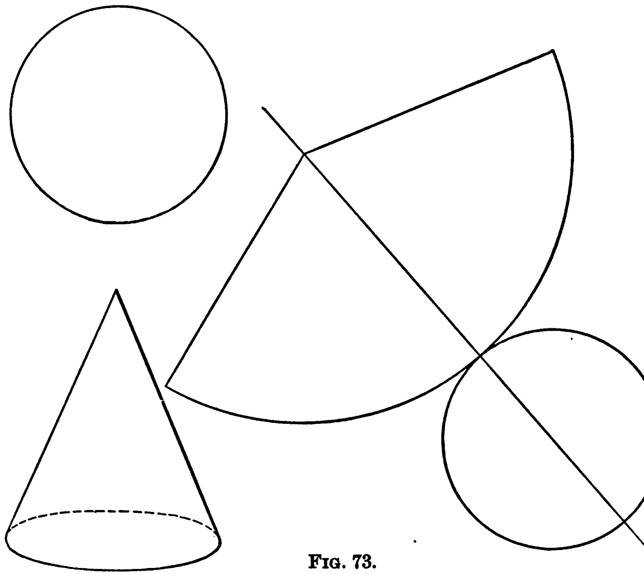


FIG. 73.

2. What is the diameter of the base of a cone made by folding a paper sector of 90°, the radius of the sector being 4 inches? What is it, if a sector of 72° is used?

The circumference of a circle $= \frac{22}{7} \times$ its diameter.

3. If you wish to make a hollow paper cone having a slant height of 6 inches and a base with a diameter of 4 inches, how many degrees should the arc of the sector contain?

Exercise 91.

1. A blacksmith, in resetting a tire for a wheel which has shrunk, uses a little wheel to measure the required length of tire and find out how much should be cut off. Show how he obtains his result.

2. A carriage wheel whose diameter is $3\frac{1}{2}$ feet made 1500 revolutions in going a certain distance. What is the distance?

3. The wheels of a bicycle are 28 inches in diameter. The front sprocket-wheel, which is connected with the pedals, has 18 sprockets, the rear one, 9. How far does this wheel move at each revolution of the pedals? If the pedals make one such revolution per second, how far does the rider travel in 1 hour? How long will it take him to make a mile? How many times per minute must the pedals revolve, if he travels 12 miles an hour?

4. How does a cyclometer measure distance? (Fig. 74.)

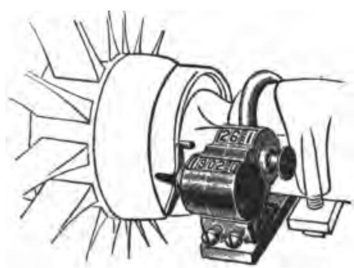


FIG. 74.

5. A bicycle is said to be "geared to 56," when it will move forward at each revolution of the pedal shaft as far as a 56-inch wheel would move forward at one revolution. How many revolutions will a 28-inch wheel make to one revolution of its pedal shaft, when it is geared to 84? to 112? What should be the ratio of the numbers of sprockets on the two sprocket-wheels in each case?

6. Find the gear of a 28-inch wheel, if the number of sprockets is:

On the Front Sprocket-wheel.		On the Rear Sprocket-wheel.				
		(i)	(ii)	(iii)	(iv)	(v)
(*a*)	16	7	8			
(*b*)	18	6	7	8	9	
(*c*)	20	5	7	8	10	
(*d*)	30	5	6	7	8	10

7. An arc of a circle is 66 inches long. The angle which it subtends at the centre is 45°. Find the radius.

8. Find the length of an arc of an angle of 45° in a circle with a radius of 35 inches.

9. If in a certain circle an arc of 120° is 75 feet long, what is the length of an arc of 40° in a circle whose radius is 3 times that of the first circle?

10. How large in degrees is an arc which, in length, is equal to the radius of the circle?

11. How far does a point on the equator turn in 1 minute on account of the rotary motion of the earth? (Radius of earth = 4000 miles.)

Exercise 92.

1. How would you determine the length of material to be used in making the scrolls in bent-iron work? (Chart VIII.)

2. How would you calculate their lengths, if you knew that they were made up of half-circles, quarter-circles, and straight lines?

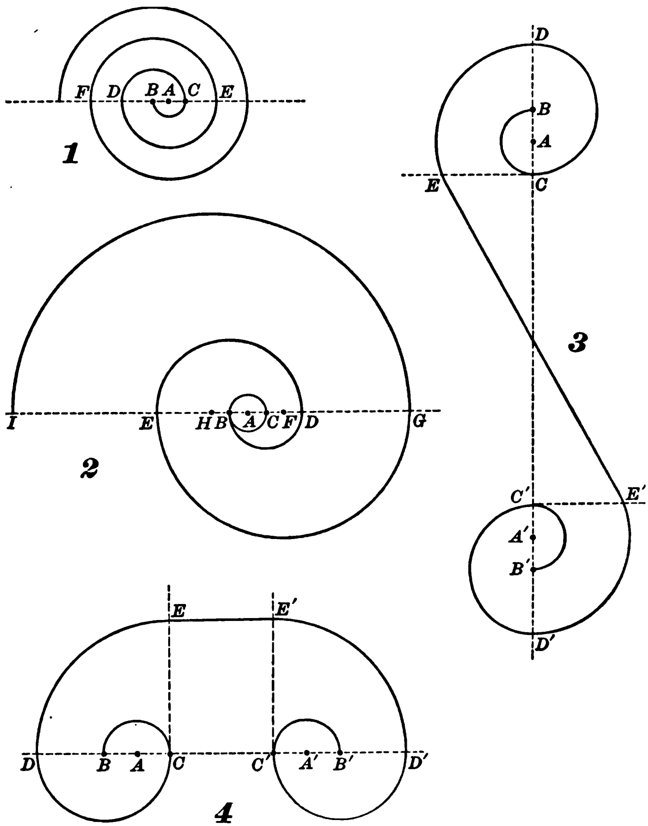


CHART VIII.

3. Determine the length of the spiral *1*, Chart VIII, if

$$AB = AC = \tfrac{3}{32} \text{ in.},$$
$$BC = BD,$$
$$AD = AE,$$
$$BE = BF.$$

4. Determine the length of the spiral *2*, Chart VIII, if

$$AB = AC = CF = BH = \tfrac{3}{32}\text{ in.},$$
$$CB = CD,$$
$$BD = BE,$$
$$FE = FG,$$
$$HG = HI.$$

5. Draw spiral *2*, when $CF = \frac{1}{2}$ of AC and $= \frac{1}{8}$ in.

6. Copy the scroll in *3*, Chart VIII, making

$$AB = AC = \tfrac{1}{4}\text{ in.},$$
$$BC = BD,$$
$$AD = AE.$$

7. Make a design for a fence, drawing five of these scrolls side by side in a row.

8. Determine length of curve in *4*, Chart VIII, if

$$AB = AC,$$
$$BC = BD, \quad \text{Scale: 1 in.} = 1\tfrac{1}{3}\text{ ft.}$$
$$CD = CE.$$

9. Draw this curve to scale: 1 in. $=$ 1 ft.

10. Numbers *1*, *2*, and *3* of Chart IX are drawn to the scale, 1 in. $=$ 8 ft. Determine the lengths of the curves represented and draw them to the scale, 1 in. $=$ 4 ft.

(*a*) In *1*, $AB = AC = EF = FG = HJ = JK,$

$$BD = DE = GI = IK,$$
$$CF = FH,$$
$$AD = DC = HI = IJ.$$

(*b*) In *2*, $AB = AC = DE = EF = \frac{3}{16}$ in.,

$$BC = CD = DF.$$

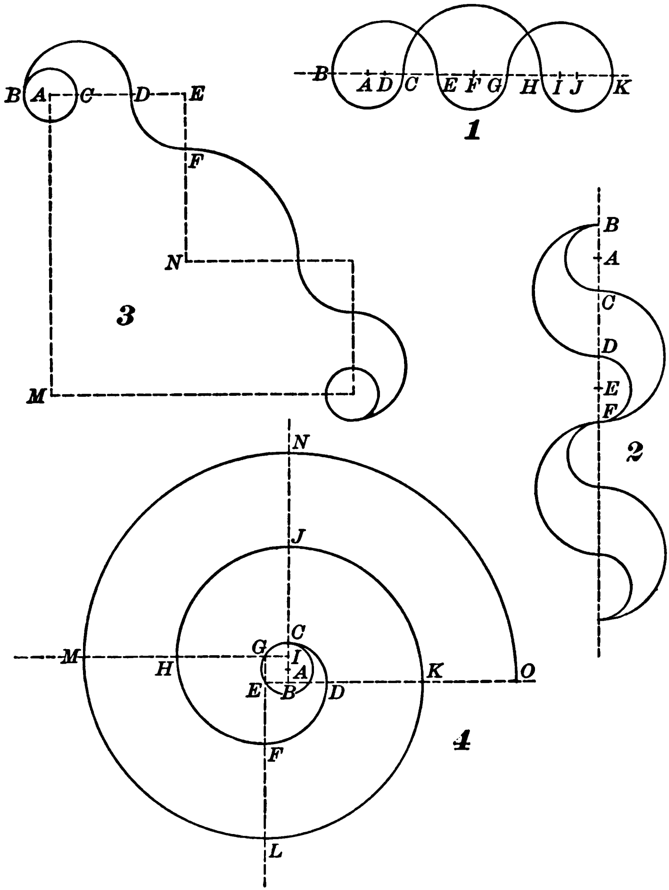


CHART IX.

(c) In *3*, $AB = AC$,

$$BC = CD = DE = EF,$$

$$NF = NF'.$$

11. Determine the length of the spiral in 4, Chart IX,

if $GIBE$ is a square

and

$$AI = AB = IC,$$
$$BC = BD,$$
$$ED = EF,$$
$$GF = GH,$$
$$IH = IJ,$$
$$BJ = BK,$$
$$EK = EL.$$

Scale: 2 in. = 1 in.

Exercise 93.

1. At what time of day is the shadow cast by an upright rod the shortest? What is its direction? Why is the shadow cast by this rod at noon-time not the same length every day? When is it longest? Shortest?

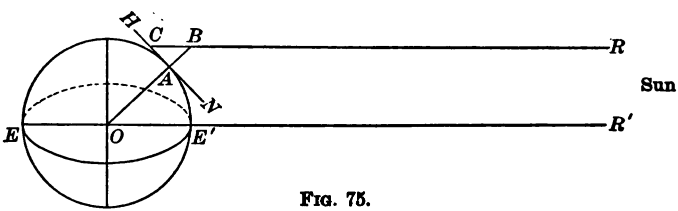


FIG. 75.

2. On March 21, a tower at A (Fig. 75), equal in height to AB, will cast a shadow equal in length to AC. What angle measures the elevation of the sun? (RB and $R'E'$ in Fig. 75 represent two parallel rays of the sun.)

3. What is the size of $\angle ACB$ (Fig. 75), if $AB = 100$ inches and $AC = 89$ inches?

SUGGESTION. See problem 17, Exercise 77, page 81.

(*a*) How large is $\angle ABC$?

(*b*) What relation exists between $\angle ABC$ and $\angle AOE'$?

SUGGESTION. See Exercise 43, page 46.

The angle AOE' corresponds to the arc $E'A$, and arc $E'A$ is called the *latitude* of the point A. For meaning of the word *latitude*, see page 8.

4. What is the latitude of point A in problem 2? If the circumference of the earth is 25,000 miles, how far is A from the equator?

5. The earliest known measurement of the circumference of the earth was made by Eratosthenes, librarian at Alexandria, Egypt, about 200 B.C. He had learned that at noon on June 21 the sun stood directly over a certain deep well

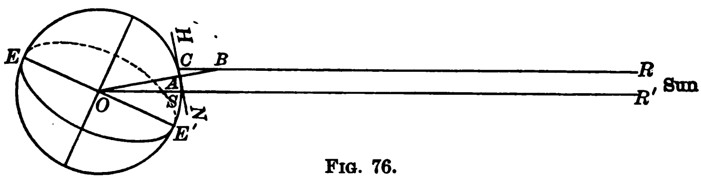


FIG. 76.

at Syene (S in Fig. 76), 500 miles directly south of Alexandria (A), and so measured the angle of elevation of the sun ($\angle ACB$ in Fig. 76) at that time in Alexandria, finding it to be $82\frac{4}{5}°$.

(*a*) How many degrees are in the arc AS?

(*b*) Show how he could calculate the circumference of the earth from these measurements.

(*c*) What is the latitude of Syene?

(*d*) What is the latitude of Alexandria?

(*e*) When would the angle ABC measure the latitude of A?

6. How can you calculate the latitude of the place you live in? Its distance from the equator?

CHAPTER IX.

AREA.

Exercise 94.

1. How many cent-pieces ($\frac{3}{4}$ of an inch in diameter) can be placed in rows, touching each other, on a table 2 feet by 3 feet? Will the surface of the table be entirely covered?

2. Figure 77 represents a small piece of ground which is to be covered entirely with grass. Which shape of sod will it be most convenient to use, the circular, triangular, rectangular, or square? How will you lay the sods? Draw a picture showing the arrangement in rows. Count the rows and the number of pieces in each row.

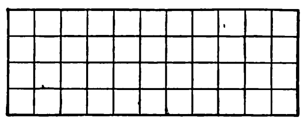


FIG. 77.

3. How many sods, each 8 inches square, will be required to sod a yard 30 feet long and 15 feet 4 inches wide?

4. How many bricks 4 inches by 8 inches are needed to lay a cellar floor 24 feet by 32 feet?

5. To find the number of tiles it would take for a floor, what would you have to know about the floor and the tiles? For what shape of tile will the figuring be the simplest?

6. How many squares, 1 inch on a side, *i.e.* how many square inches, will it take to cover a paper 10 inches long and 8 inches wide?

Finding the number of squares it will take to cover a surface is called "finding the *area* of the surface."

Exercise 95.

1. Draw a square with a side of 7 inches and divide it into square inches. Count the rows and the number of square inches in each row. What, then, is the area of the square?

2. Show by a drawing like Fig. 77 that a rectangle $4\frac{1}{2}$ inches by $2\frac{3}{8}$ inches contains $2\frac{3}{8}$ rows of $4\frac{1}{2}$ square inches each. What is its area?

3. What rule can you give for computing the area of a rectangle when you know its base and its altitude?

4. Give the area of a square with a side of

(*a*) $2\frac{1}{2}$ inches. (*b*) 3.4 cm.

5. Give the area of a rectangle

(*a*) $3\frac{1}{2}$ inches by $5\frac{1}{4}$ inches. (*b*) 15.3 cm. by 12.5 cm.

6. In what unit of square will you express the area of:

(*a*) A blackboard? (*c*) A farm?
(*b*) The floor of a room? (*d*) A garden?

7. What will it cost to paint the floor of a room 25 feet by 32 feet, at 22 cents a square yard?

8. What will it cost to cement a cellar floor 40 feet by 24 feet 9 inches, at 85 cents a square yard?

9. What is the cost of paving a court $34\frac{1}{2}$ yards wide and $61\frac{3}{4}$ yards long, at $5.25 a square yard?

(*a*) At the same rate, what would be the cost of paving a street which is 40 feet wide and 2 miles long?

10. What is the area of actual playing ground in:

(*a*) A double tennis-court?
(*b*) A basket-ball field?
(*c*) A football field?

Exercise 96.

1. Cut out two rectangles (2 inches by 1 inch). Place them with their long sides on a straight line. Then, beginning at the upper right-hand corner, cut from one of the rectangles a triangle, and fit the detached piece to the left side of the corresponding remainder in a way so as to retain its position with respect to the straight line (Fig. 78). Examine the resulting figures and state:

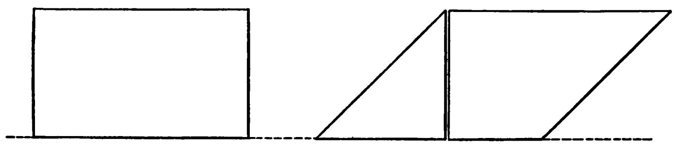


FIG. 78.

(*a*) What they are, and give your reasons.

(*b*) What their relation is to the rectangles and to each other.

(*c*). In what parts they agree and in what parts they differ.

They have equal bases and equal altitudes.

2. Can you show, by cutting, that a parallelogram is equal to a rectangle?

3. Show by a drawing how you can transform a parallelogram into a rectangle.

4. How can you compute the area of a parallelogram? What parts do you need to know?

5. Two parallelograms with the same altitude are placed side by side on the same straight line. Draw the rectangle which is equal in area to their sum.

Exercise 97.

1. A rectangular piece of land, *ABCD* (Fig. 79), is 800 feet long and 400 feet wide. A railroad, *EFGH*, is run through it obliquely, the distances, *EF* and *GH*, measuring 50 feet each. How much land is taken by the railroad company? What is the rest of the land worth, at $ 500 an acre?

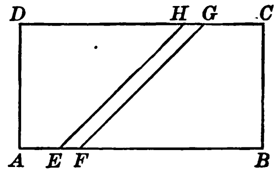


Fig. 79.

2. Divide a square into

 (*a*) Four equal squares.
 (*b*) Six equal rectangles.
 (*c*) Eight equal right triangles.

3. Divide a rectangle into three equal rectangles.

4. Divide a parallelogram into six equal parallelograms.

5. Change a parallelogram, *MNOP*, whose $\angle M = 60°$, to one whose $\angle M = 45°$.

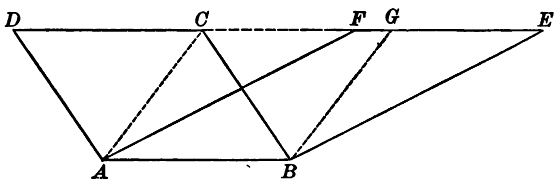


Fig. 80.

6. In Fig. 80, what relation exists between the areas of the parallelograms *ABEF* and *ABCD*? How can you show this by cutting?

Hint. Begin by drawing *ABCD*. Form *ABGC*. Then change *ABGC* to *ABEF*.

Exercise 98.

1. Cut out two triangles, ABC, equal in size and form. Place them side by side as indicated in Fig. 81, $\angle C$ adjacent to $\angle B$. How do the opposite lines run? Why? What kind of a figure is formed? Compare it with triangle ABC as to bases, altitudes, and areas.

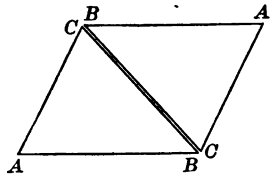


FIG. 81.

2. Can *every* triangle be seen as half of some parallelogram? How will you prove it?

SUGGESTION. Prove it for any triangle selected at random.

3. Show by cutting or drawing how to transform a triangle into a parallelogram. Into a rectangle.

4. Show that the area of a triangle is equal to one-half of a rectangle with equal base and equal altitude. In how many ways, then, can you compute its area?

See note on page 31.

Exercise 99.

1. Find the area of a triangle, ABC,* having given:

(*a*)	$BC = 15\frac{1}{2}$ yards and	$AL = 5\frac{1}{4}$ yards.
(*b*)	20 ft. 5 in.	9 feet 3 inches.
(*c*)	25 cm.	16 cm.
(*d*)	3.5 cm.	2.8 cm.

2. Find BL' in (*a*), if $CA = 12$ yards.

3. Find CL'' in (*b*), if $AB = 10$ feet 6 inches.

4. Find BL' in (*c*), if $CA = 18$ cm.

* See Exercise 37, page 41.

5. Find the area of a right triangle, having given:

(a) $AB = 3.6$ cm. and $BC = 4$ cm.
(b) 108 feet 81 feet.

Exercise 100.

1. If you place, on the same straight line, a number of triangles, having equal bases and equal altitudes, what must be true of the position of their vertices? Can you draw one straight line through all their vertices? Why? How can you show that two triangles, having equal bases and equal altitudes, are equal in area?

See Exercise 98.

2. Divide a triangle into three equal triangles.

See Exercise 75.

Are these parts equal both in form and size? If so, what should you be able to do with them?

See Exercise 65.

3. If the area of a triangle is 3 square inches and the base is 2 inches, what is the altitude? How many triangles satisfying these conditions can you draw?

4. Draw a triangle equal to the sum of two triangles having equal altitudes.

HINT. Begin by placing the two given triangles with their bases continuous on the same straight line.

5. Construct a rectangle equal to the sum of two triangles having equal altitudes.

Exercise 101.

1. Make a paper trapezoid $ABCD$, in which AB and DC are the parallel sides. Draw EF connecting the middle points of AD and BC. Cut along EF and fit the two parts in the same way as indicated in Fig. 32, page 49. What is the resulting figure? Change it to a rectangle.

2. Show that the area of a trapezoid is equal to one-half of a rectangle which has the same altitude and a base equal to the sum of the parallel sides of the trapezoid.

See problem 3, Exercise 45, page 49.

3. Can you calculate the areas of the triangles *ACD* and *ABC*, when you know *AB*, *CD*, and *CE*?

4. Find the area of a trapezoid, *ABCD* (Fig. 82), having given:

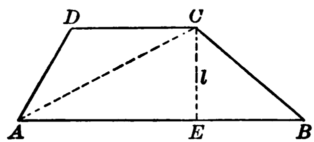


FIG. 82.

	AB	*DC*	*CE*
(*a*)	15 cm.	7 cm.	9 cm.
(*b*)	38 feet	22 feet	15 feet.
(*c*)	1 foot, 2 inches	9 inches	6 inches.

5. Find the altitude of a trapezoid, if the area is 775 square feet and the parallel sides are 42 feet and 20 feet, respectively.

Exercise 102.

1. How would you determine the area of a figure shaped like a kite (Fig. 83)?

2. Draw any polygon. Show how its area may be computed from the triangles into which it may be divided. Indicate the parts that need to be known in each of the triangles.

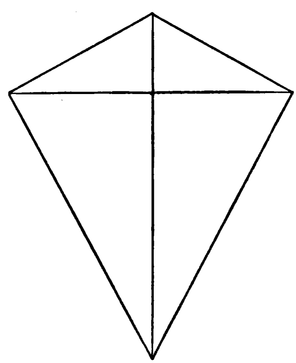


FIG. 83.

3. Compute from the given measurements the areas of:

(*a*) I, Chart X. (*c*) III, Chart X.
(*b*) II, Chart X.

4. Compute the areas of V and VI on Chart X, measuring the lines which are necessary for your purpose. Scale: 1 inch = 80 rods.

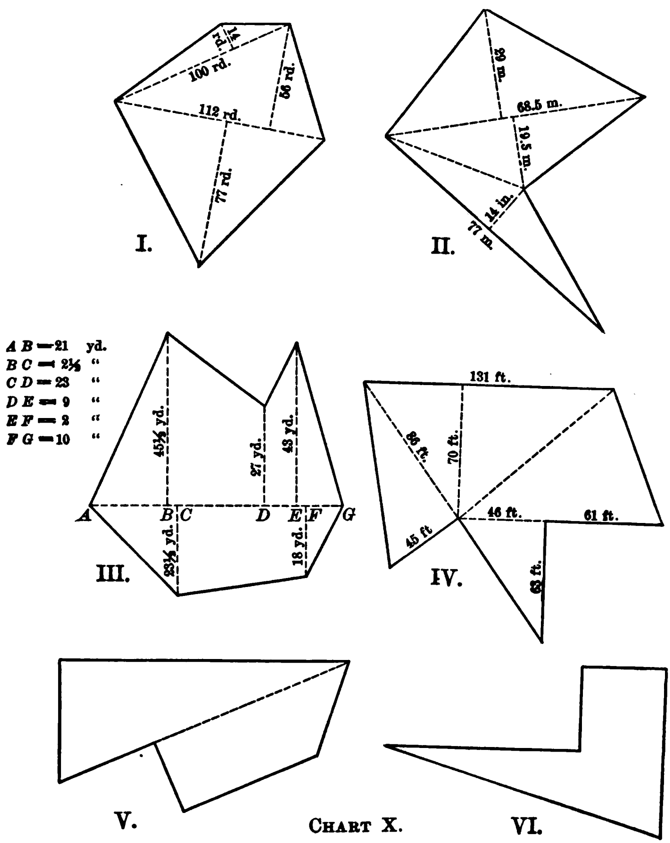


CHART X.

5. (*a*) Select any three-sided field, as *ABC* (Fig. 84). Measure the three sides *AB*, *BC*, and *CA*. (Surveyors also measure some other line, *CD*, as a sort of check or proof-line.) Draw to a suitable scale. From this plat then determine the altitude *CE* and compute the area.

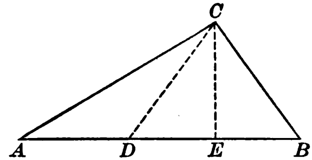


FIG. 84.

(*b*) Do likewise with a four-sided field.

(*c*) Do likewise with your school yard.

Exercise 103.

1. What is the least number of diagonals needed to divide a quadrilateral into triangles? A pentagon? A hexagon? An octagon? A decagon?

2. Make a hexagon and draw from one of its vertices as many diagonals as you can. What relation is there between their number and the number of sides of your polygon? How do you account for it?

Then draw as many diagonals as you can from the other vertices. What is the total number of diagonals in the hexagon? Why is this number not six times the number of diagonals drawn from one vertex?

3. Find by construction (i) the number of diagonals which you can draw from each vertex and (ii) the total number of diagonals in:

(*a*) A polygon with five sides.
(*b*) A polygon with seven sides.
(*c*) A polygon with nine sides.
(*d*) A polygon with *n* sides.

4. How can you find the total number of diagonals in a polygon without making a diagram? Verify your answer on Fig. 85.

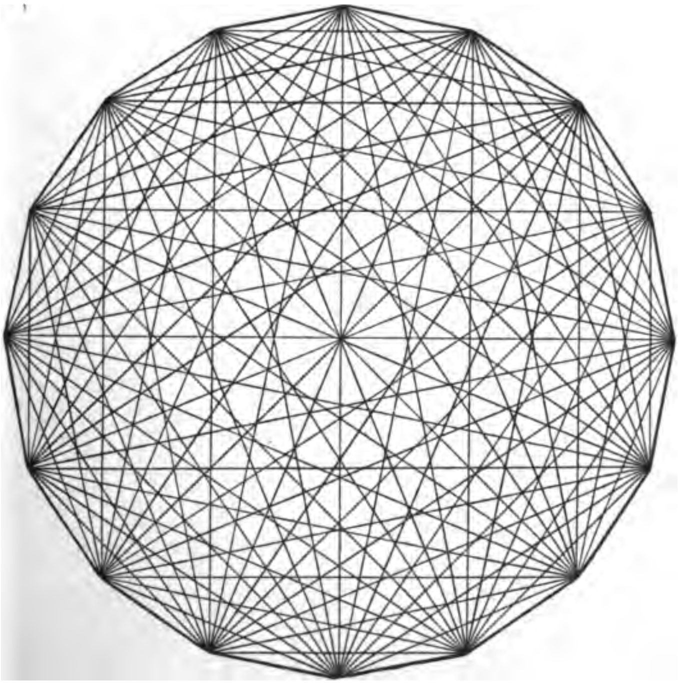


FIG. 85.

5. Without making a diagram, find the number of diagonals that can be drawn

(*a*) In a polygon of twelve sides.
(*b*) In a polygon of twenty sides.
(*c*) In a polygon of forty-eight sides.
(*d*) In a polygon of n sides.

Exercise 104.

1. Draw, to a convenient scale, a line AB 50 feet long. From any point on this line, and in opposite directions, erect perpendiculars, one 10, the other 15, feet long. Join their extremities with A and B, and find the area of the quadrilateral thus formed.

2. One diagonal of a quadrilateral of unequal sides is 42 rods. The altitudes of the triangles forming the quadrilateral and having the diagonal for a base, are 28 and 21 rods, respectively. Find the area of the quadrilateral.

3. Surveyors sometimes obtain the area of a piece of ground, — especially when the diagonals cannot conveniently be measured, — by laying off a rectangle around it, thus:

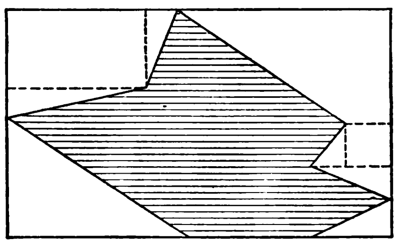


FIG. 86.

Why will this give the correct result? What are the advantages of this method?

4. Make use of this method to find, approximately, the area of your state. Consult your maps.

Exercise 105.

To find the area of a regular hexagon.

1. In Chart XI you will find a regular hexagon divided into:

(*a*) Four triangles.
(*b*) Two rectangles and four right triangles.
(*c*) Two trapezoids.
(*d*) Three rhombuses. (*e*) Six equilateral triangles.

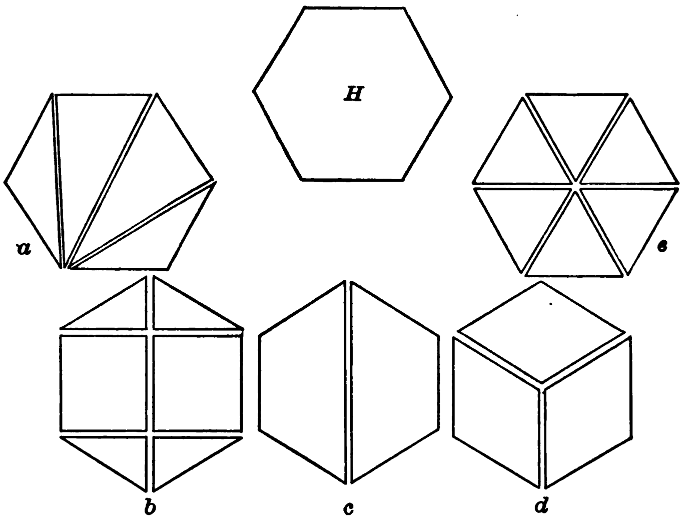


CHART XI.

Which of these divisions is the best for the purpose of computing the area of a regular hexagon? Why?

2. Divide a regular hexagon into six equilateral triangles — division (*e*). Place these triangles side by side on a

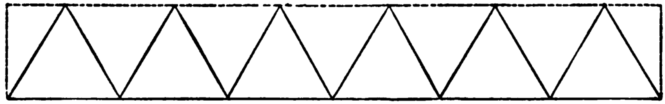


FIG. 87.

straight line (Fig. 87), draw the line which passes through their vertices, and complete the rectangle enclosing them.

(*a*) What is the relation of this rectangle to the sum of the triangles? Why?

(*b*) What line corresponding to the altitude of this rectangle can you draw in your hexagon?

This distance of the middle point of a side of a regular polygon from the centre is called the *apothem*. (See Exercise 62, page 65.)

(*c*) What lines of the hexagon make up the base of this rectangle?

The sum of the sides which bound a polygon is called its *perimeter*. Perimeter means *measure around*.

(*d*) Why is the area of *every* regular hexagon equal to one-half of such a rectangle?

Exercise 106.

1. Make a regular pentagon and show that its area is equal to one-half of a rectangle which has a base equal to the perimeter and an altitude equal to the apothem of the pentagon.

2. Show the same for a regular pentagon. A regular octagon.

Make a statement about the area of any regular polygon.

3. Find the area of a regular hexagon with a side equal to 1 inch and an apothem approximately equal to $\frac{7}{8}$ inch.

4. Inscribe a regular nine-sided figure in a circle of 2-inch radius, using the protractor. Measure (*a*) its apothem, (*b*) one of its sides, and (*c*) calculate its area.

5. Do likewise with

(*a*) A regular octagon. (*b*) A regular decagon.

6. Find the area of a regular octagon with a side equal to $\frac{3}{4}$ inch and an apothem equal to $\frac{15}{16}$ inch.

Exercise 107.

To find the area of a circle.

Observe how closely the perimeter of a polygon like the one in Fig. 85, page 119, resembles the circumference of a circle, and consider how much more like a circle the figure would be, if there were a still greater number of sides. For this reasou, we conclude to treat the circle as a regular polygon of very many and very small sides, having an apothem approximately equal to the radius. (See Chart XII.)

Considering the circle as a regular polygon of many small sides, show by cutting that its area is approximately equal to one-half of the area of a rectangle having the circumference of the circle for its base and the radius of the circle for its altitude. (Chart XII.)

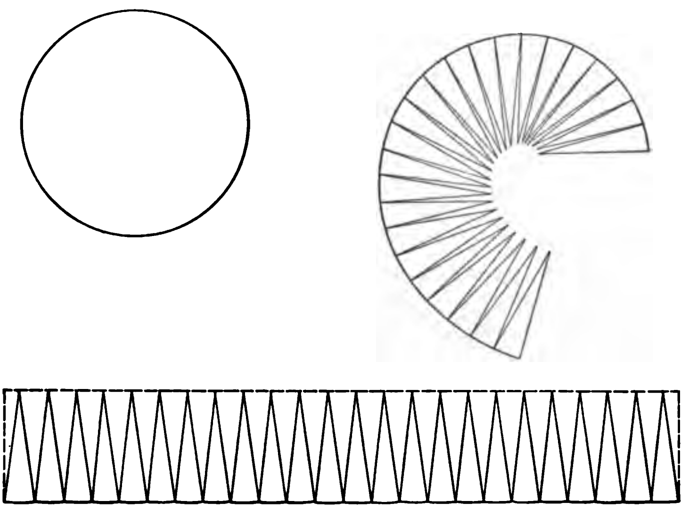


CHART XII.

1. What is the area of a circle whose circumference is equal to 11 feet?

2. A circular piece of ground, 60 feet wide, has a walk around it $3\frac{1}{2}$ feet wide. Find the area of the walk.

3. There are two concentric circles. The diameter of the inner is 15 feet 4 inches, and that of the outer is 17 feet 2 inches. Find the area of the ring.

Exercise 108.

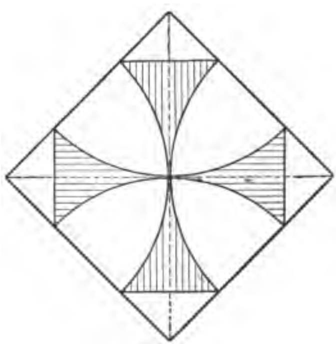


FIG. 88.

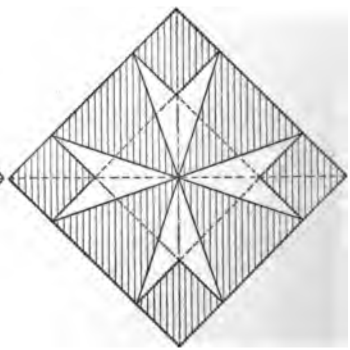


FIG. 89.

1. Figures 88 and 89 represent garden plots. The shaded parts are laid out in flowers and the rest is grass. Determine the areas of the flower beds and the lawns. Scale: 1 inch = 48 feet.

2. What is the difference in area between a circle having a radius equal to 8 inches and its regular inscribed hexagon?

3. Find the area of the cross-section of the largest square beam that can be sawed from a log $2\frac{1}{2}$ feet in diameter.

Exercise 109.

1. What is the area of a quadrilateral field, the diagonals of which intersect at right angles and are 120 and 175 rods, respectively? (Fig. 83.)

Draw through the vertices of the figure lines parallel to the diagonals, thus making a rectangle enclosing the figure.

2. Find the area of a rhombus whose diagonals are 55 yards, and 100 yards, respectively.

3. A field is bounded by four straight lines, two of which are parallel. The sum of the parallel sides is 1000 rods, and their distance from each other is 300 rods. What is the area of the field in acres?

4. A rectangular garden, 150 feet long and 90 feet wide, has a walk around it 5 feet wide. There is a walk 3 feet wide, extending lengthwise through the middle of the garden. Draw a plan. Compute the number of square feet in the walks.

5. The side of a square is 36 yards. Find the areas of the inscribed and circumscribed circles.

6. How much larger is a square than an equilateral triangle, both having a perimeter of 48 feet?

Exercise 110.

1. Construct two squares, so that the sides of one are twice as long as those of the other.

What is the ratio of:

(i) The diagonals?
(ii) The perimeters?
(iii) Any two lines similarly drawn?

How do these ratios compare with the ratios of the sides?

How often can you put the smaller square into the larger?

What, then, is the ratio of their areas? How do these ratios compare with the ratios of the sides?

2. Construct two squares, so that the sides of one are three times as long as those of the other and answer the questions given in preceding problem.

3. Construct two triangles, so that the sides of one will be one-third as long as those of the other.

What is the ratio of:

(i) The altitudes?
(ii) The perimeters?
(iii) Any two lines similarly drawn?

How do these ratios compare with the ratios of the sides?

How often can you put the smaller triangle into the larger? (Fig. 90.)

What, then, is the ratio of their areas?

How do the ratios of their areas compare with the ratios of their sides?

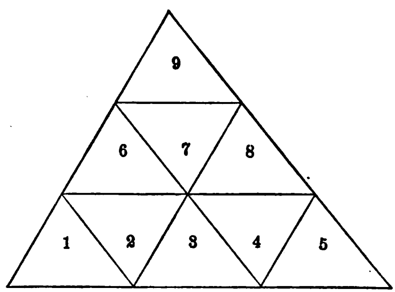


Fig. 90.

4. To what extent is the area of a regular polygon changed, if its side is reduced to one-sixth of its original length?

5. The sides of two regular hexagons are as 3 to 5. The area of the first is 60 square feet. What is the area of the other?

Exercise 111.

To enlarge a map in a given ratio, we spread over it a network of equal figures, usually squares or rectangles as in *A* of Fig. 91, and then draw the outline of the map in a larger network of figures similar to the first, as in *B* of Fig. 91.

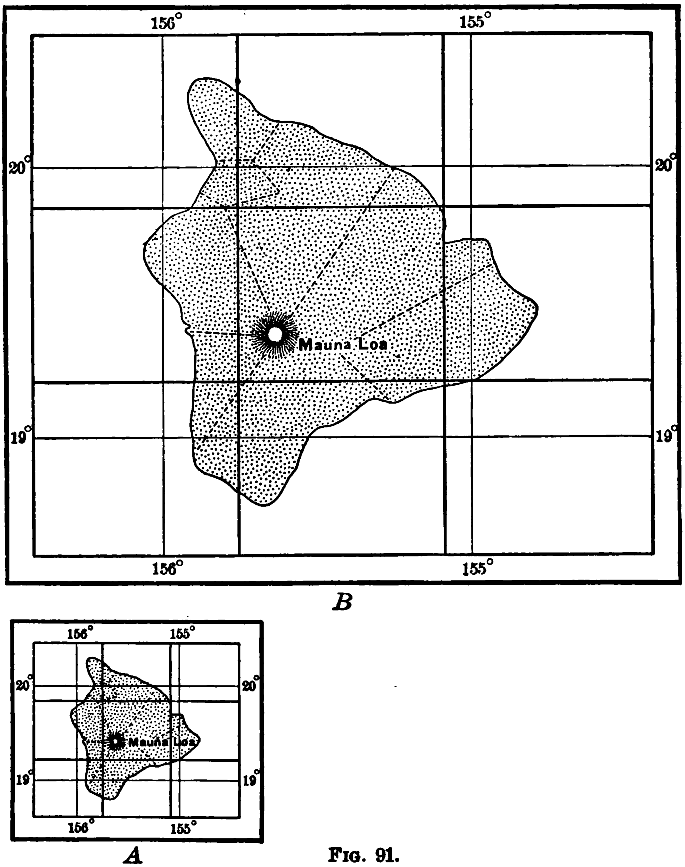


FIG. 91.

1. Study the two maps in Fig. 91. What is the relation of the sides of the rectangles in A to those in B?

What is the ratio of their areas? What is the ratio of the lines of the maps passing through parts of the network that are similarly placed? To what scale, then, is the larger map drawn?

How many times as large as A is B?

2. What would have to be the ratio of the sides of the figures in a network to enlarge a map 25 times? 100 times?

3. A map is 2 feet long and 1 foot wide. To what scale is it drawn, if it represents 20,000 square miles of surface?

CHAPTER X.

THE RIGHT TRIANGLE.

Exercise 112.

1. Cut a 3-inch paper square, $ABCD$. Join A and C and locate a point F on AC 1 inch from C. Then draw EG and HI through F and parallel, respectively, to DC and CB.

See Fig. 92.

(*a*) What kind of a figure is $AIFE$? $FGCH$? $IBGF$? $EFHD$?

(*b*) What relation exists between $IBGF$ and $EFHD$? Why?

SUGGESTION. Fold your paper over AC as an axis.

Now cut HF and GF nearly to F, taking care not to separate the small square $FGCH$ entirely from the rest of the paper. Then turn the rectangles $EFHD$ and $IBGF$ under, folding along the lines EF and FI, respectively. This will leave only the two squares, $AIFE$ and $FGCH$, visible.

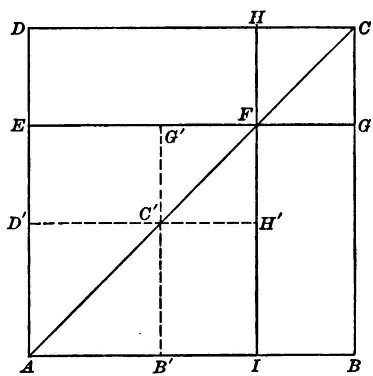


FIG. 92.

(*c*) What relation exists between these two squares taken together and the original square, $ABCD$? Express this relation by filling out the blanks in

$$AIFE + FGCH = ABCD - (\text{———} + \text{———}).$$

2. Cut another 3-inch square, *ABCD*, and on its diagonal *AC* locate the points *F* and *F'*, *F* 1 inch from *C*, and *F'* 1 inch from *A*. Through each one of these points, then, draw lines parallel, respectively, to the sides of *ABCD*. Join *EK*, *HL*, *LH*, and *HE*, and folding along these lines, turn the four corners of the paper under.

See Fig. 93.

(*a*) What is the relation of the triangles *EHD*, *AKE*, *KBL*, and *LCH* to one another?

(*b*) What relation exists between $\angle HED$ and $\angle DHE$? (See problem 6, Exercise 35, page 38.) Between $\angle HED$ and $\angle AEK$? How large is $\angle KEH$?

(*c*) Why is *EKLH* a square?

(*d*) What relation exists between *EKLH* and *ABCD*? Express this relation by filling out the blanks in

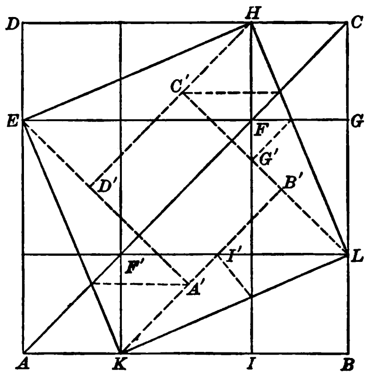


FIG. 93.

$$EKLH = ABCD - (\text{———} + \text{———} + \text{———} + \text{———}).$$

(*e*) How does the area of the four triangles turned back in Fig. 93 compare with the area of the two rectangles turned back in Fig. 92?

(*f*) What relation exists between the square *EKLH* of Fig. 93 and the squares *AIFE* and *FGCH* of Fig. 92?

Exercise 113.

1. Why must *EKLH* of Fig. 93 and *AIFE* and *FGCH* of Fig. 92, when placed together as in Fig. 94, enclose a right triangle *EFH*?

EHLK (Fig. 94) is called the square on the hypotenuse; *AIFE*, the square on one side of the right angle; and *FGCH*, the square on the other.

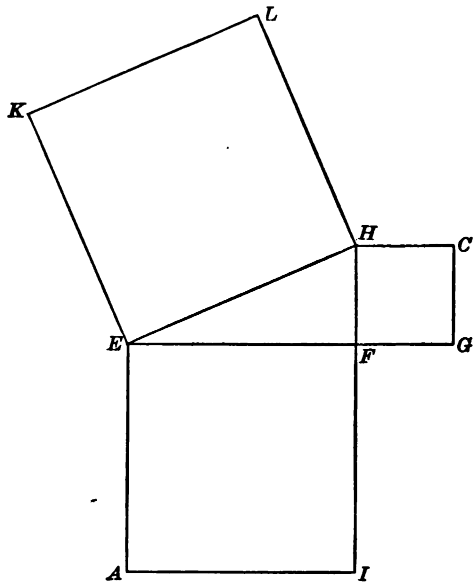


Fig. 94.

What is the relation of the squares on the sides of the right triangle, to that on the hypotenuse, in Fig. 94? If this relation existed in *every* right triangle, how would you word a statement to that effect?

2. What size must you cut your paper squares to show this relation for a right triangle, the sides of which are:

(*a*) 1 inch and 2 inches?
(*b*) $2\frac{1}{2}$ inches and 3 inches?
(*c*) $1\frac{3}{4}$ inches and $2\frac{1}{2}$ inches?

3. Make any right triangle and show, by paper-folding, that the square on the hypotenuse is equal to the sum of the squares on the sides of the right angle. Why may we now conclude that this relation holds for every right triangle?

The discovery of this relation is credited to Pythagoras, a famous Greek mathematician, who lived about 540 B.C. It is

thought that he was led to make it by observing the tiles in the floor of one of the public baths so noted in his time. Figure 95 shows a portion of tile floor with one tile missing and the others around it loose, a condition which, it is not at all improbable, presented itself to the eyes of Pythagoras on one of his visits to the baths.

4. What kind of a triangle is represented by the missing tile? (Fig. 95.) Show that the square on the hypotenuse is equal to the sum of the squares on the sides. (Count the tiles.) Why does this not prove the same relation for every right triangle?

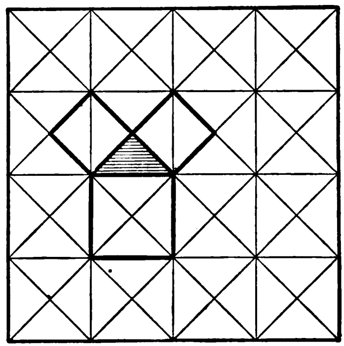


FIG. 95.

5. Make two squares with sides of 3 and 4 inches, respectively, and draw the square which will be equal to their sum.

Exercise 114.

1. A kite became fastened in the top of a tree. The kite-string was 5 rods long and the distance from the point where the string touched the ground to the bottom of the tree was 4 rods. How high was the tree?

2. Find the length of one of the equal sides of an isosceles triangle which has an altitude equal to 3 inches and a base equal to 8 inches.

3. The hypotenuse of a right triangle is 85 cm.; and the sides of the right angle are as 3 to 4. Find the sides and the area.

4. Two vessels start at the same time from the same place, and sail one due west and the other due south, at the rates of $4\frac{1}{8}$ and $5\frac{1}{2}$ miles per hour, respectively. How far apart will they be at the end of 8 hours? Draw to a convenient scale.

5. The diagonals of a rhombus are 60 and 80 yards, respectively. Find each side and the altitude. Test your answer by drawing to a scale.

6. The hypotenuse of a right triangle is 128 feet long, and the two sides of the right angle are equal to each other. Find their lengths.

7. A man drove two stakes, A and B, 12 links apart, and fastened to them the ends of a 24-link chain. At the end of which link from A should he stretch this chain to C, so that AC will be perpendicular to AB? Why?

How to make a right angle by stretching a cord around three pegs, 3, 4, and 5 units apart, was known to the Egyptians as early as 2000 B.C.

8. How is it possible to measure the height of a room with a pole that is longer than the room is high?

Exercise 115.

1. Fold a parallelogram so as to divide it into two unequal parallelograms and two equal parallelograms. Give reasons for the equality of the latter. (See Exercise 112.)

2. What relation exists in Fig. 93 between

   (*a*) $A'B'C'D'$ and $ABCD$?
   (*b*) $A'B'C'D'$ and the squares on EF and FH?

3. Construct a square equal to the sum of two given squares.

4. Construct a square equal to the difference of two given squares.

5. (*a*) By how much does the square on the sum of two straight lines exceed the sum of the squares on the lines?

(*b*) How much is the square on the difference of two lines less than the sum of the squares on the lines?

6. Construct a square whose area is equal to (*a*) 13 square inches, (*b*) 5 square inches, (*c*) 24 square inches, (*d*) 7 square inches, (*e*) 11 square inches.

Note that $13 = 9 + 4$ or $3^2 + 2^2$.

7. Construct a square equal to the sum of three given squares.

8. Determine the areas of all the squares, rectangles, and triangles in Figs. 92 and 93 for the following values of EF and FH:

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)
EF =	5 in.	7 in.	10 in.	60 mm.	45 mm.	9 yd.	a	m
FH =	2 in.	3 in.	2 in.	20 mm.	30 mm.	4 yd.	b	n

INDEX.